

Broadband Vibrational Energy Harvesting using Nonlinear Systems

D. Mallick, S. Roy

Tyndall National Institute
Cork, Ireland

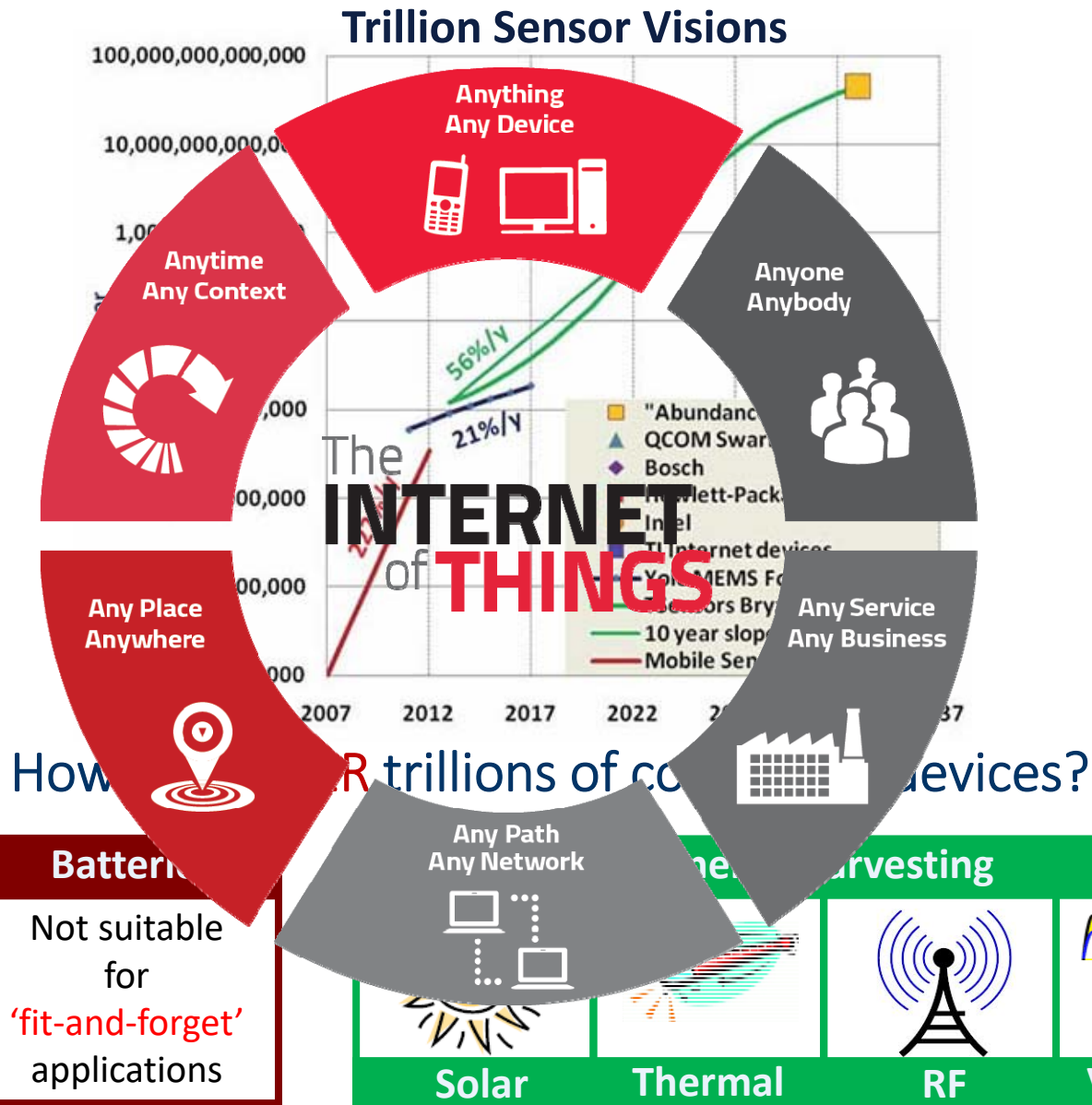
EnerHarv 2018

PSMA Inaugural International Energy Harvesting Workshop
May 29-31, Cork, Ireland



Internet of Things (IoT)

Wirelessly connecting **Anything**



Mechanical Energy Harvesting

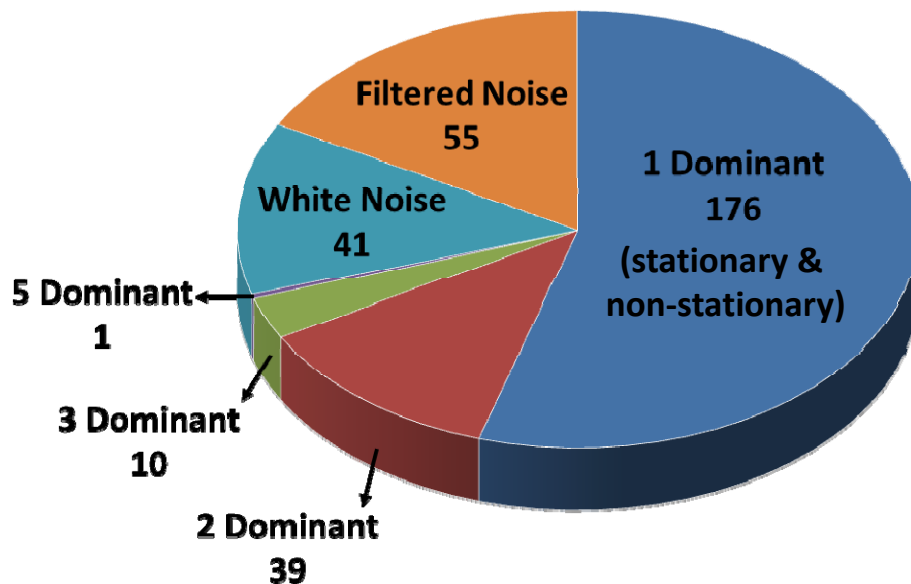
What releases mechanical energy – **Almost Anything**



- Online database – **Real Vibrations** - NiPS lab, University of Perugia, Italy.
 - **EH Networks Database** – EPSRC Funded Network.
- Vibration data from more than 600 sources comprising –
 - ❖ Human Activities
 - ❖ Machine
 - ❖ Automobile/Aerospace
 - ❖ Large Structures

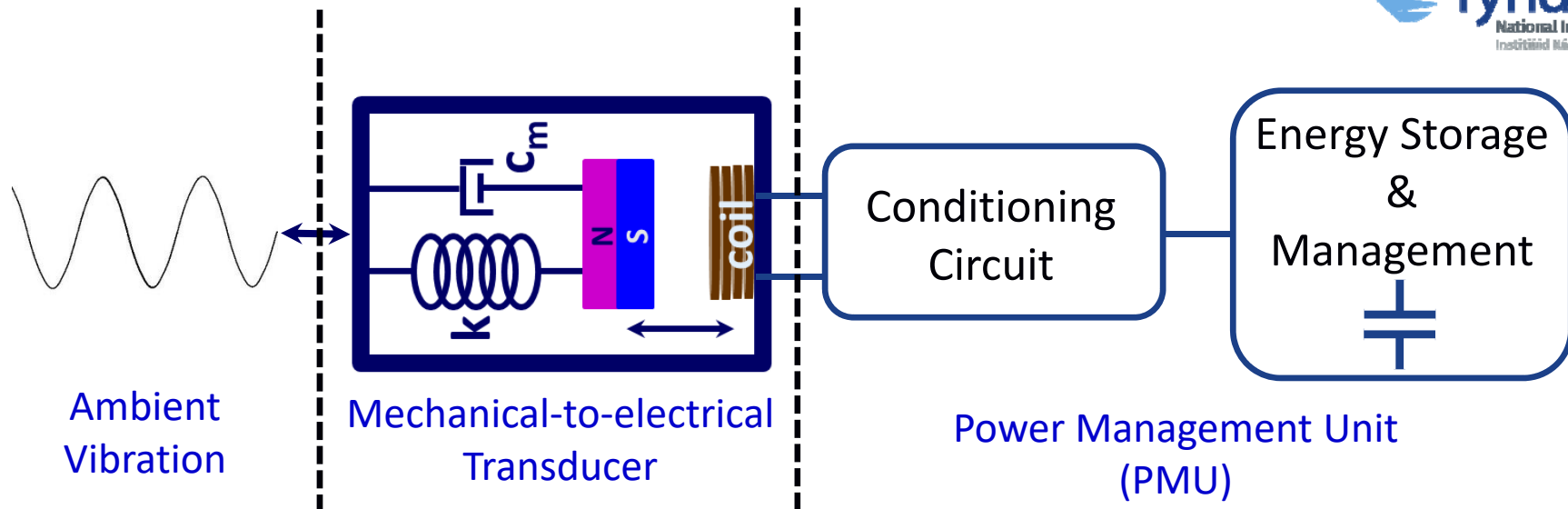
Mechanical Energy Harvesting

What releases mechanical energy – **Almost Anything**



- 23% Sources – single dominant, stationary frequency.
- 53% Sources contains –
 - ❖ single dominant, non-stationary frequency
 - ❖ multiple dominant, stationary frequencies
 - ❖ white/filtered noise - broadband

Inertial Energy Harvesting System



Second Order Spring Mass Damper System

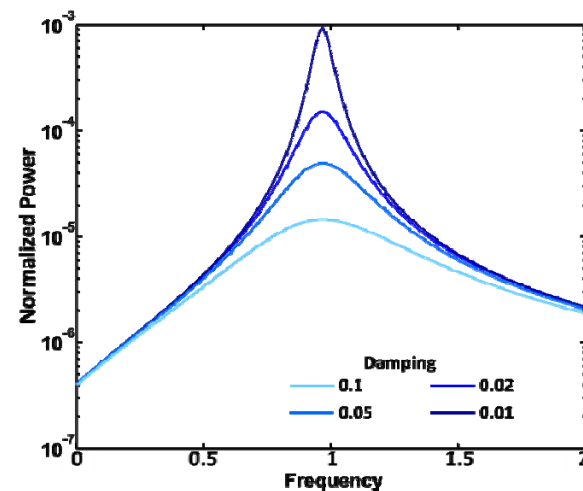
$$m\ddot{x}(t) + (c_m + c_e)\dot{x}(t) + kx(t) = -m\ddot{y}(t)$$

Power dissipated in the electrical damper

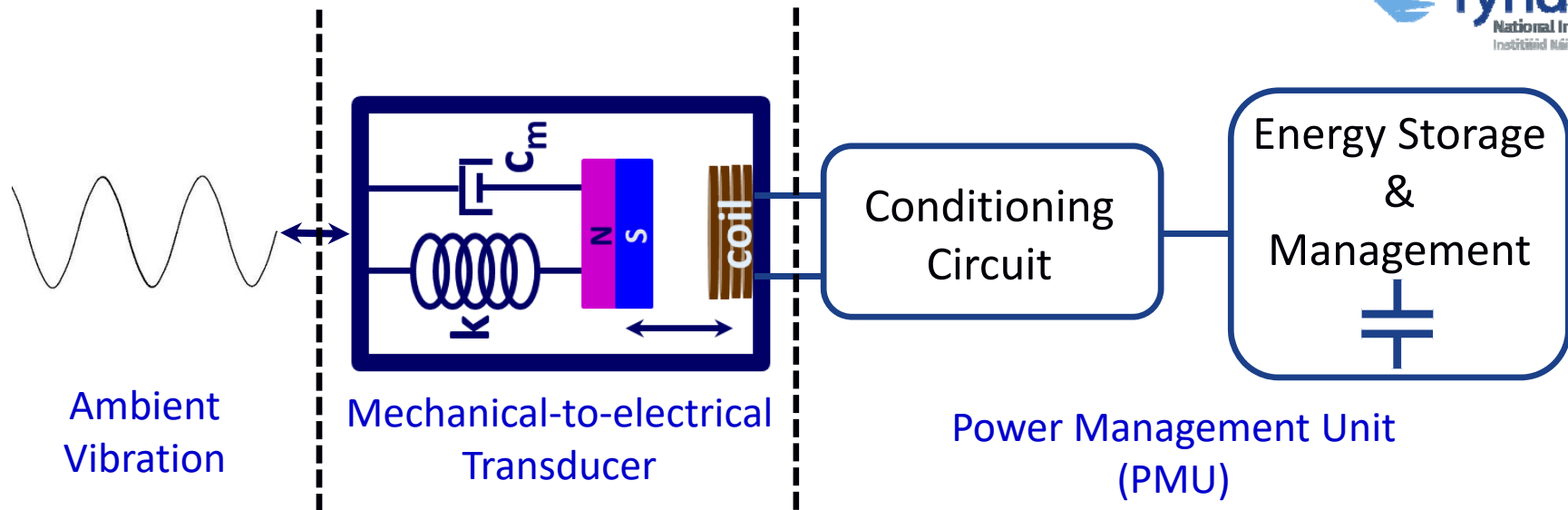
$$P_{\text{elec}} = \frac{c_e \left(\frac{\omega}{\omega_n}\right)^3 Y_0^2 \omega^3}{2\omega_n [\{2\rho_T \left(\frac{\omega}{\omega_n}\right)\}^2 + \{1 - \left(\frac{\omega}{\omega_n}\right)^2\}^2]}$$

At Resonance:

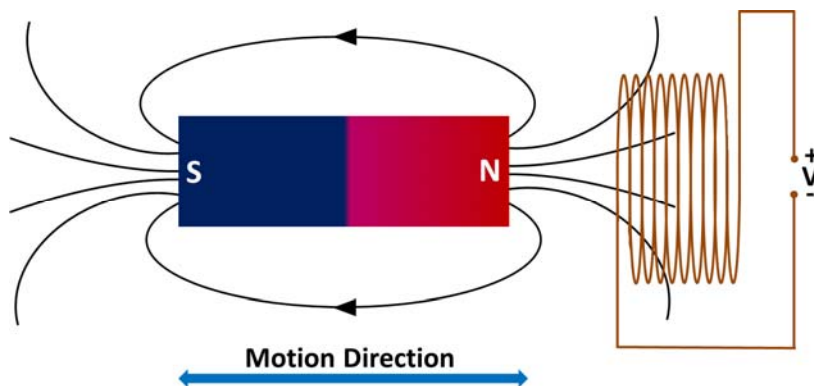
$$P_{\text{elec}}|_{\omega=\omega_n} = \frac{c_e m^2 Y_0^2 \omega_n^4}{2(c_m + c_e)^2}$$



Inertial Energy Harvesting System



Principle governing EM generators is Faraday's Law



$$EMF (V) = -N \frac{d\Phi}{dt}$$

$$\text{Induced voltage } V_o = Z_{max} \omega_n N \frac{d\Phi}{dz}$$

$$\text{Load Voltage } V_L = V_o \frac{R_L}{(R_L + R_C)}$$

$$\text{Power delivered to load } P_L = \frac{V_L^2}{2R_L}$$

Wideband Energy Harvesting

Target: Wideband Operation

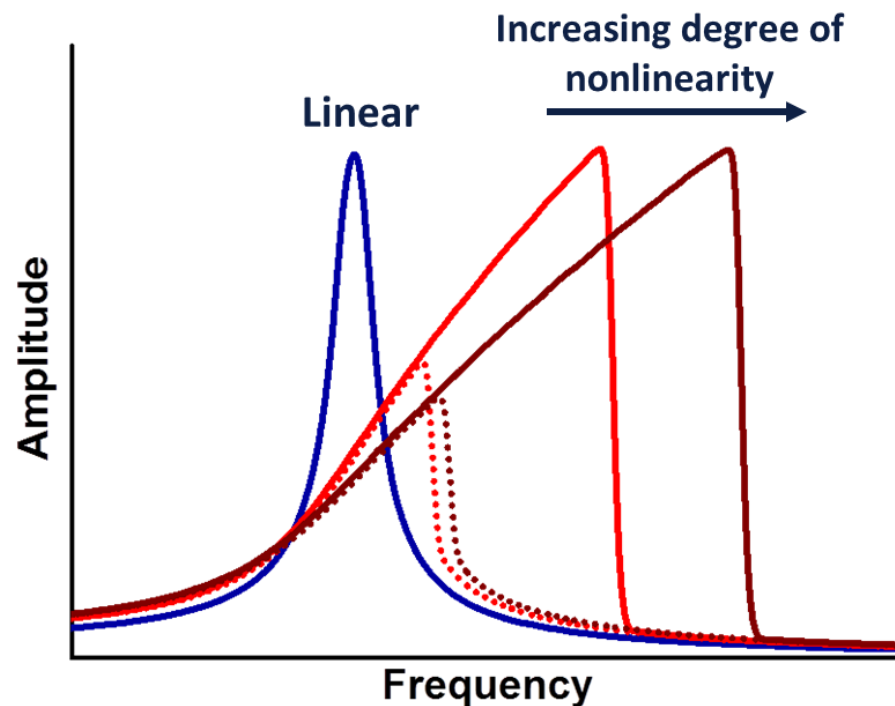
Linear  Nonlinear

Equation of motion of simplest energy harvesting systems

$$m\ddot{x}(t) + (c_m + c_e)\dot{x}(t) + F(x) = -m\ddot{y}(t)$$

Linear $F(x) = kx$

Nonlinear $F(x) = kx + k_n x^3$

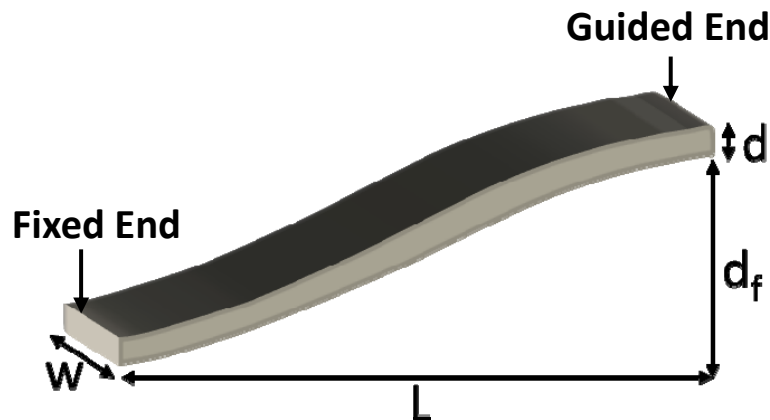


- Nonlinearity through stiffness
- Increase in Bandwidth
- No tuning needed; No array of devices needed
- Gain in output power

$$\frac{P_{nonlinear}}{P_{linear}} \leq \frac{4}{\pi}$$

Analysis of Nonlinear Stretching

Clamped Guided MEMS beam:

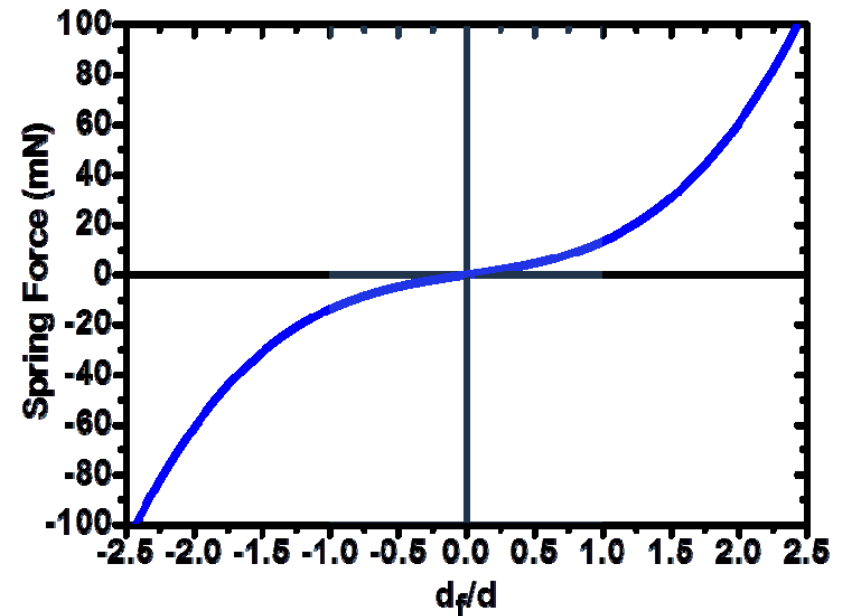


Total spring reaction force:

$$F = F_b + F_s = -\frac{Y}{L^3} W d^4 \left[\frac{d_f}{d} + \frac{18}{25} \left(\frac{d_f}{d} \right)^3 \right]$$

Bending Force
Stretching Force
Cubic Nonlinearity

- ❖ $\frac{d_f}{d} \ll 1$ - nonlinearity is insignificant
- ❖ $\frac{d_f}{d} \geq 1$ - cubic nonlinearity is significant



linear stiffness constant: $k = \frac{Y W d^3}{L^3}$

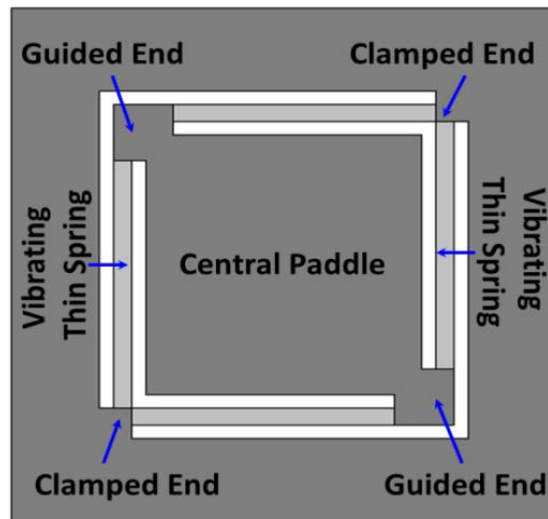
nonlinear stiffness constant: $k_n = \frac{18 Y W d}{25 L^3}$

$$\frac{k_n}{k} = \frac{18}{25 d^2}$$

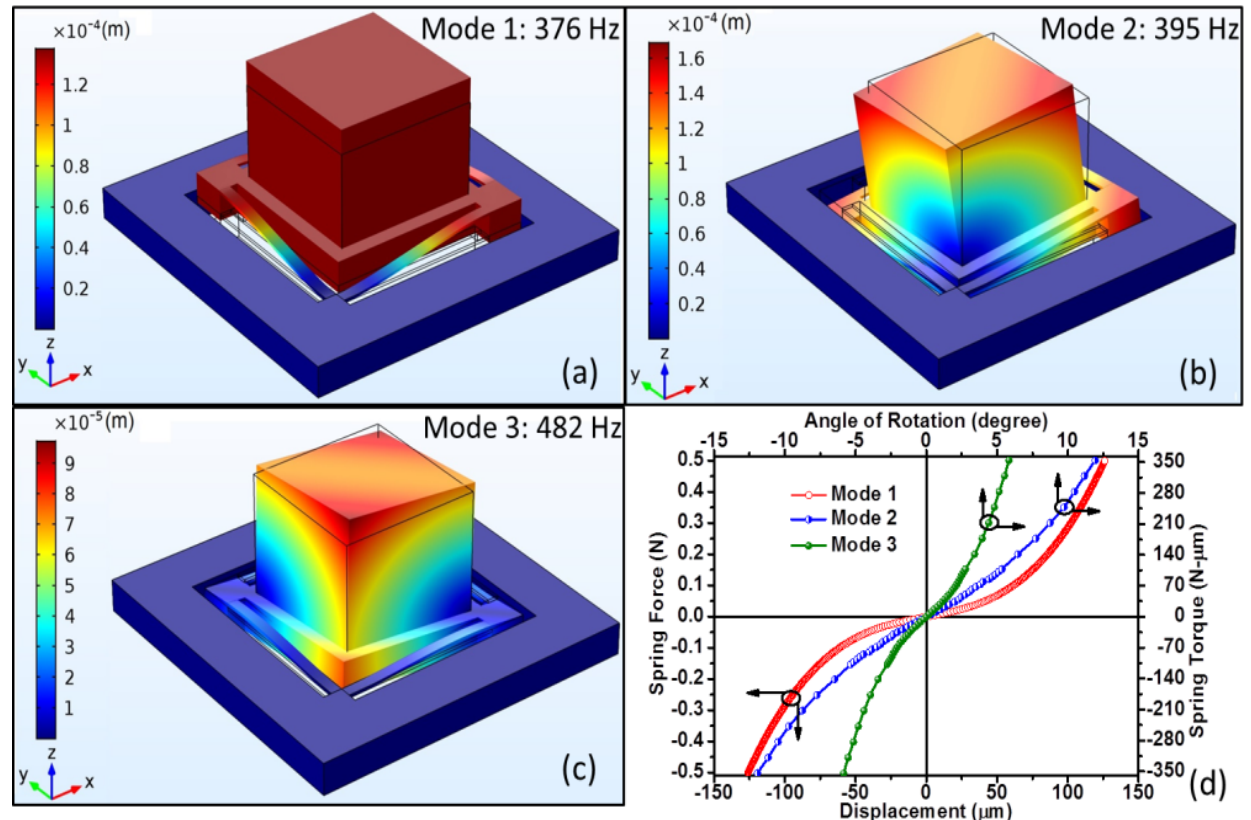
Smaller the thickness, larger the nonlinearity

MEMS Nonlinear Wideband Operation

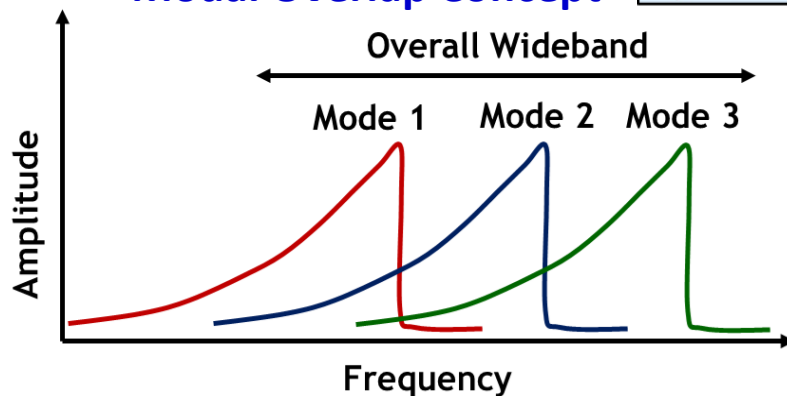
Design of the Nonlinear Architecture



FEM Simulation



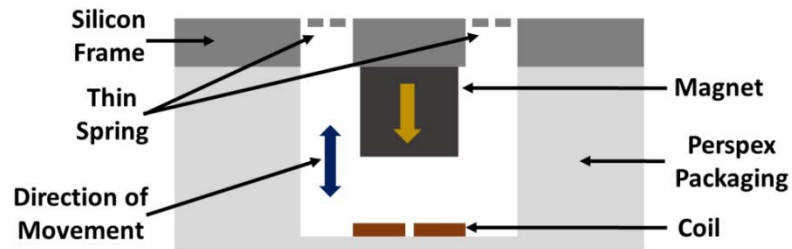
Modal Overlap Concept



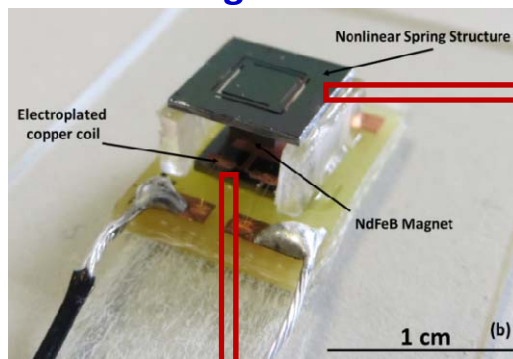
- Nonlinear effect observed in first 3 modes
- Closely spaced mode overlap in frequency response

Device Fabrication and Testing

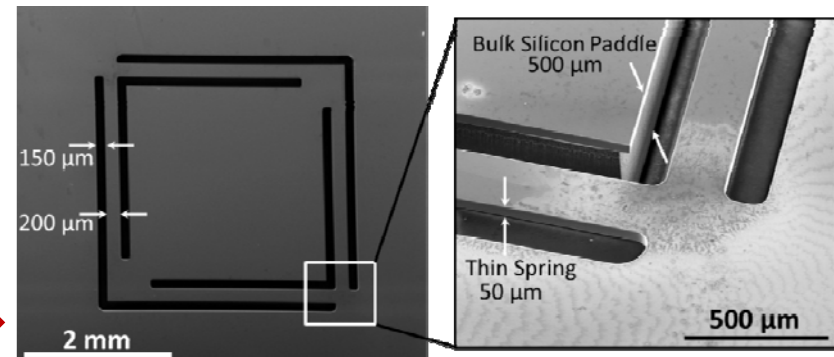
Device Schematic Cross-section



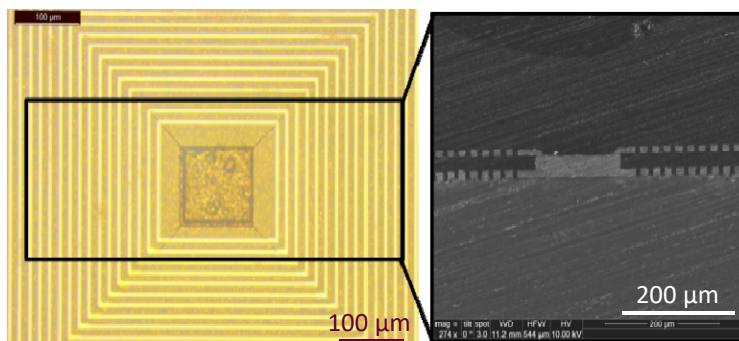
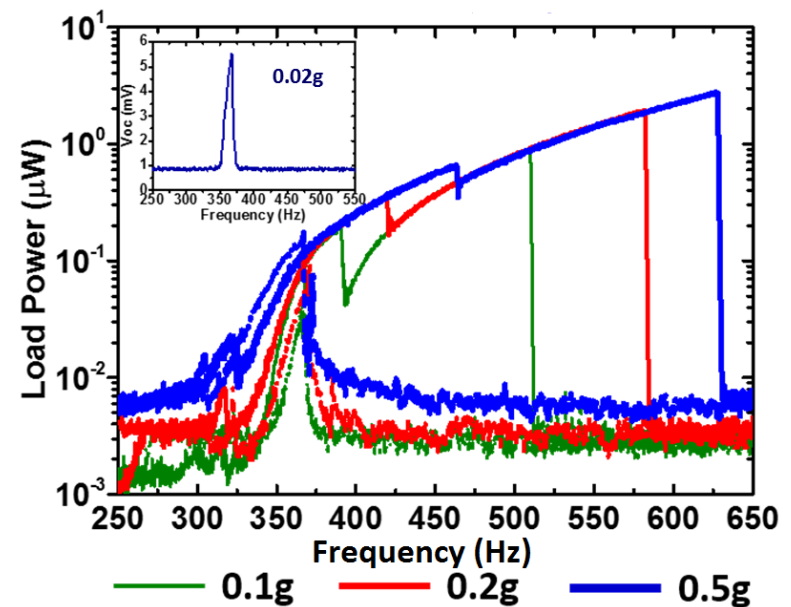
Packaged Device



ICP- DRIE etched SOI spring structure



Load Power vs Frequency Response

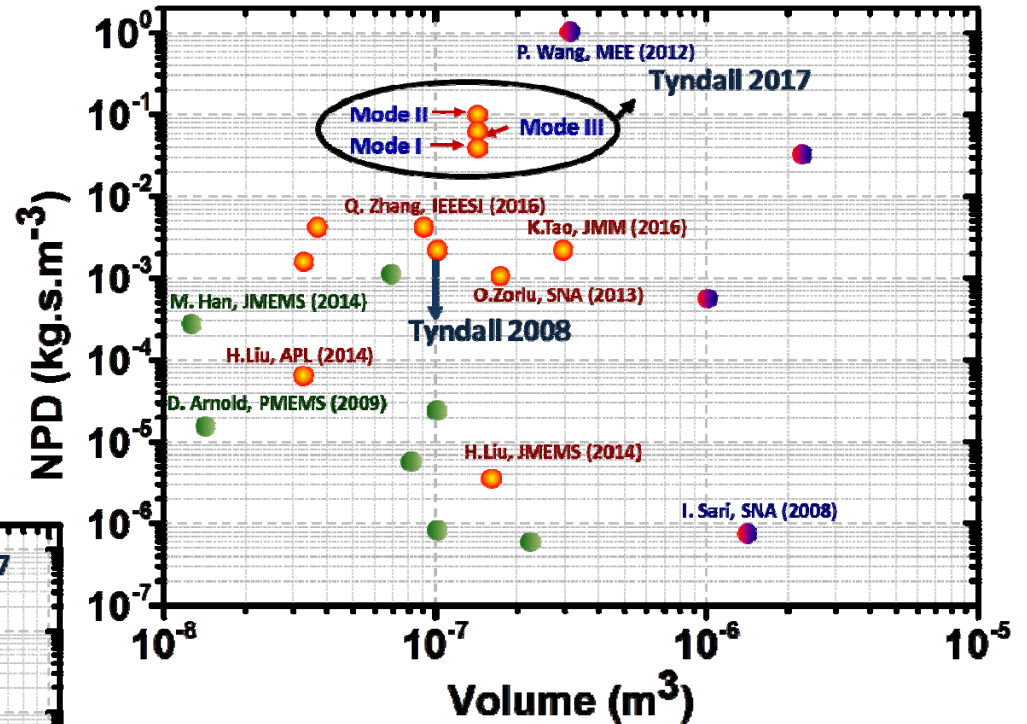
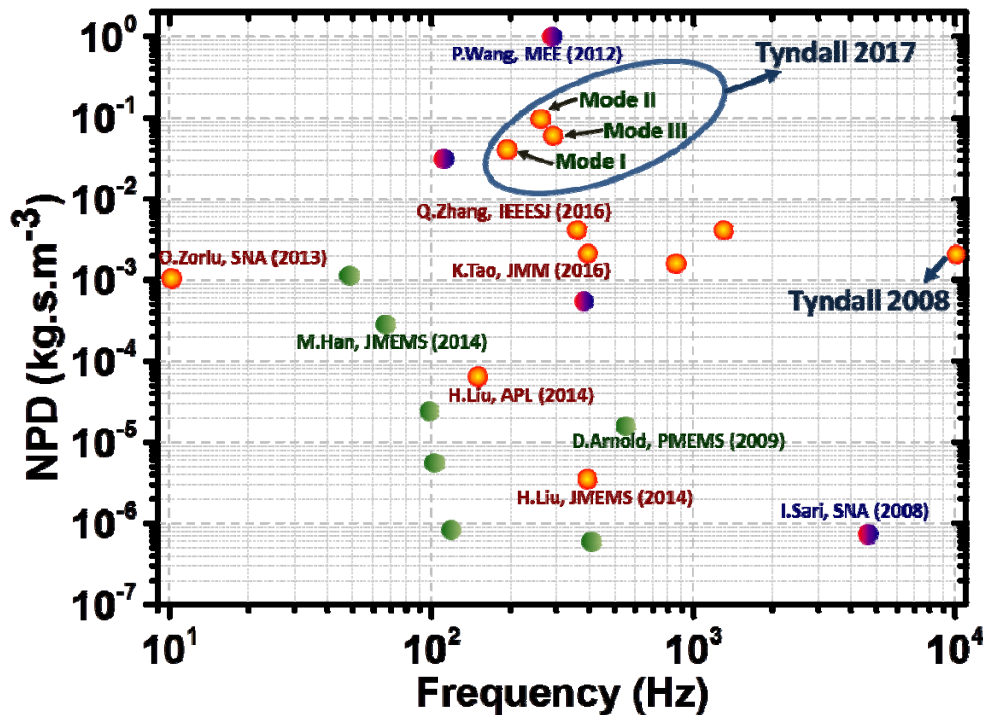


Electroplated Double Layer Copper Coil

- Bandwidth – 82 Hz @ 0.5g with $2.8 \mu\text{W}$

Benchmarking the MEMS EM VEH

$$\text{Normalized Power Density (NPD)} = \frac{\text{Power}}{\text{Acceleration}^2 \cdot \text{Volume}}$$



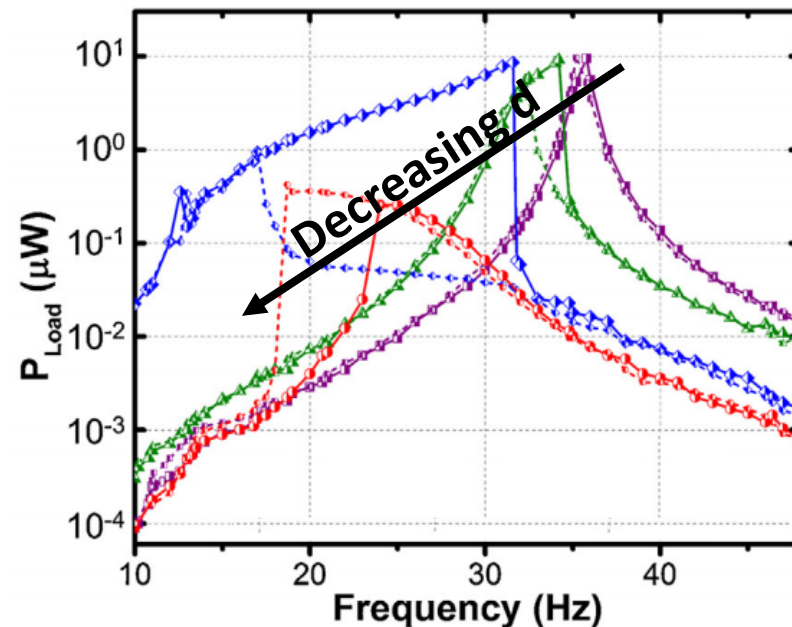
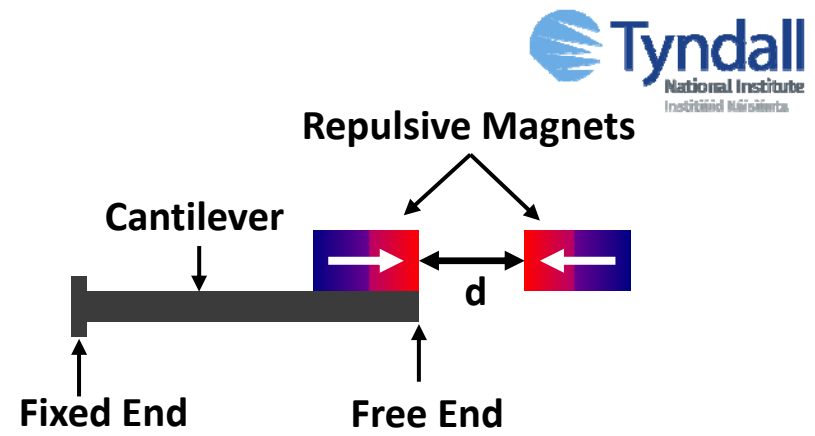
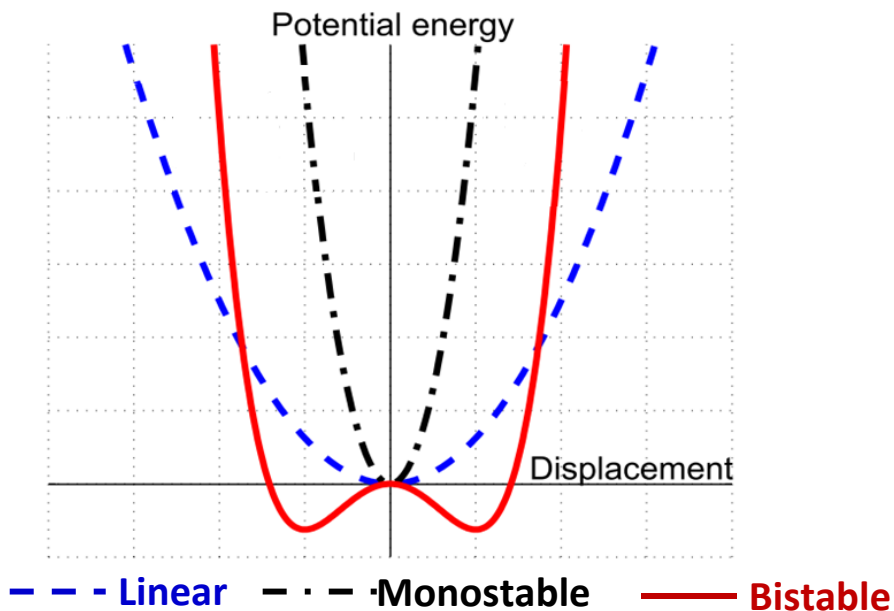
- MEMS Suspension only
- MEMS Suspension with micro-coil
- Fully MEMS integrated

Bistable Energy Harvesting

Potential Energy:

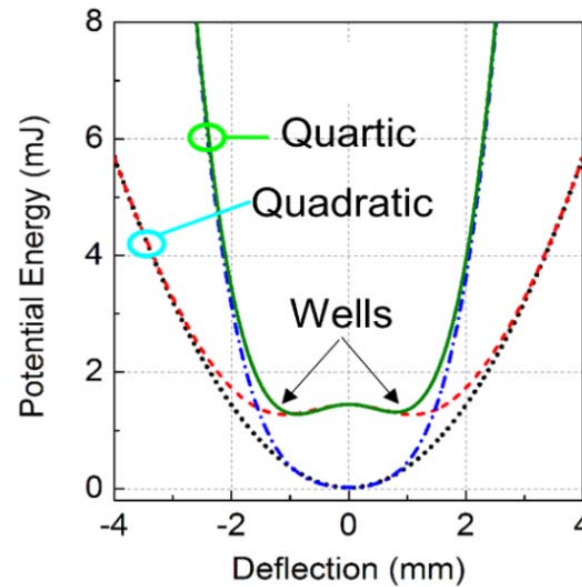
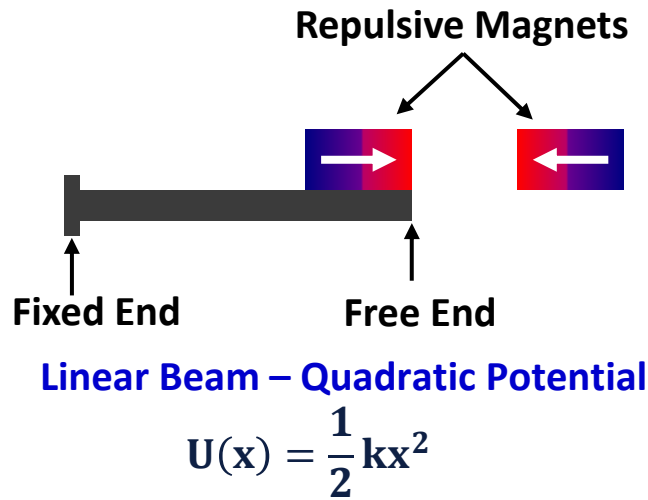
$$U(x) = \frac{1}{2}kx^2 + \frac{1}{4}k_n x^4$$

- $k_n = 0, k > 0$: linear
- $k_n > 0, k > 0$: monostable
- $k_n > 0, k < 0$: bistable



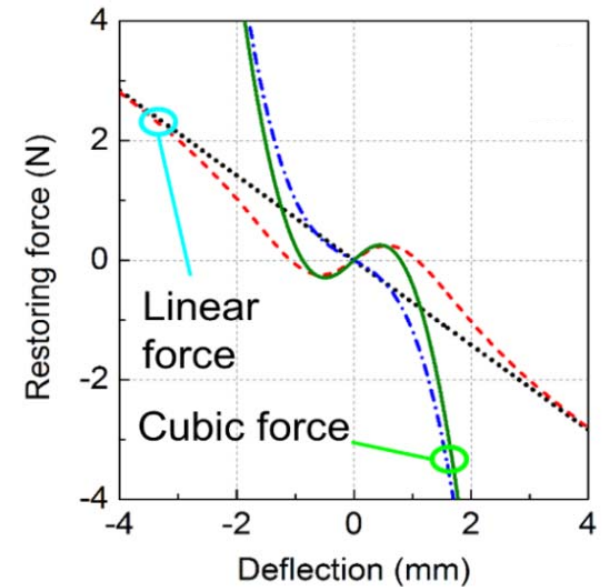
- Magnetic repulsive force creates bistable potential
- Intra-well oscillation – monostable like behaviour
- Inter-well oscillation – more interesting

Effect of Fundamental Potentials



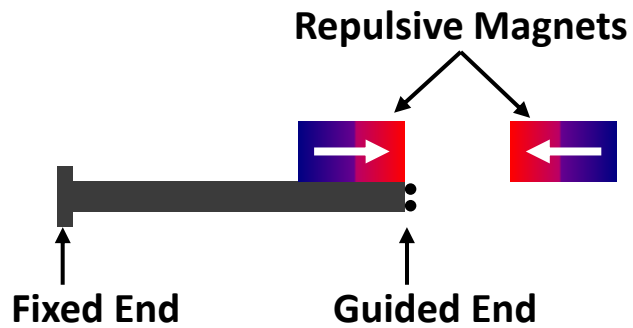
..... Linear

----- Bistable



- . - . Monostable

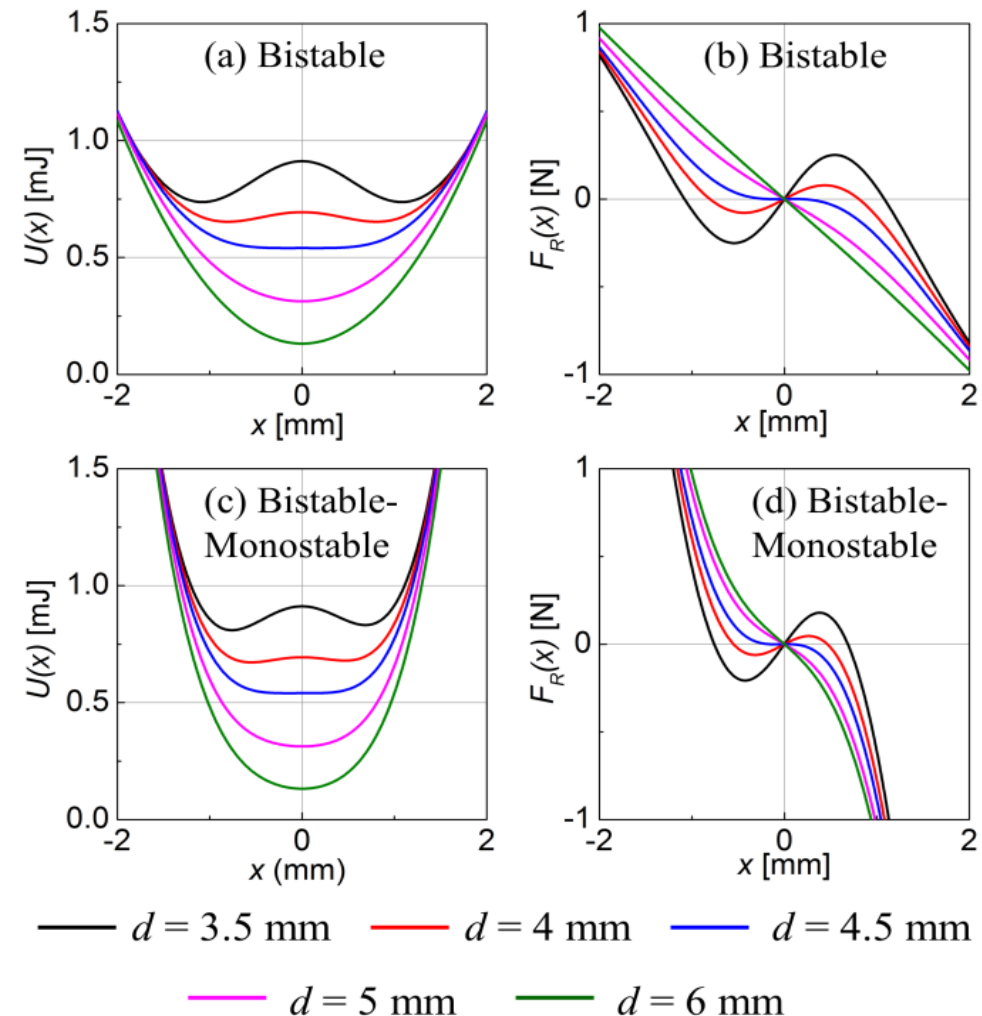
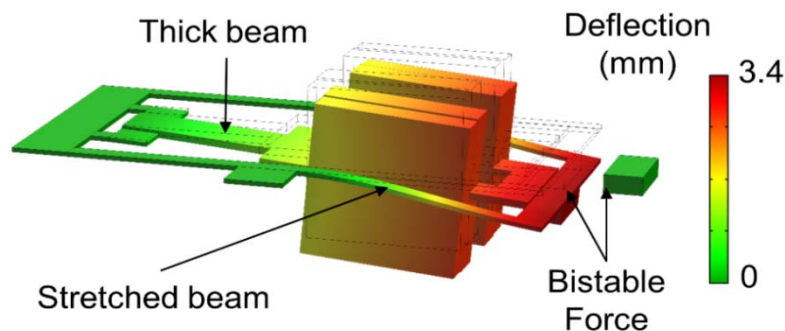
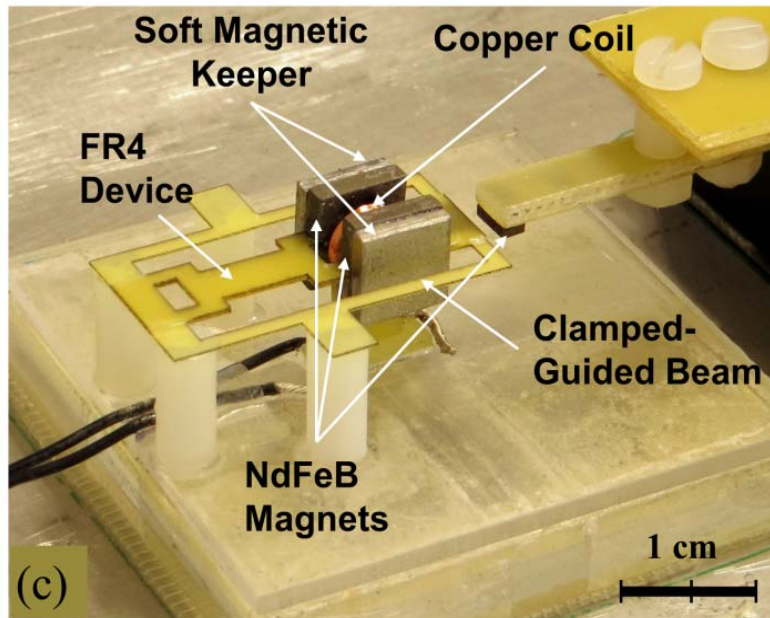
———— Bistable - Monostable



Nonlinear Beam – Quartic Potential

$$U(x) = \frac{1}{2} kx^2 + \frac{1}{4} k_n x^4$$

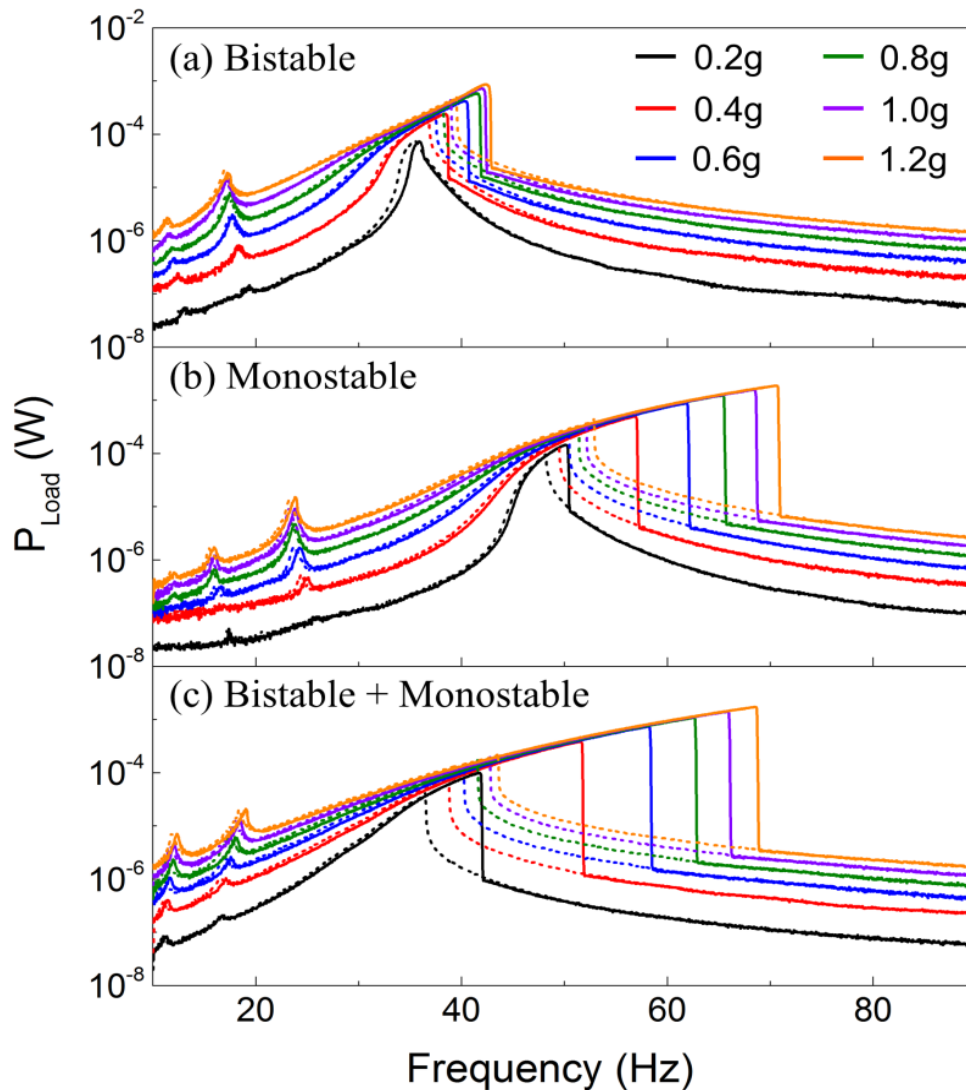
Combined Multiple Nonlinearity



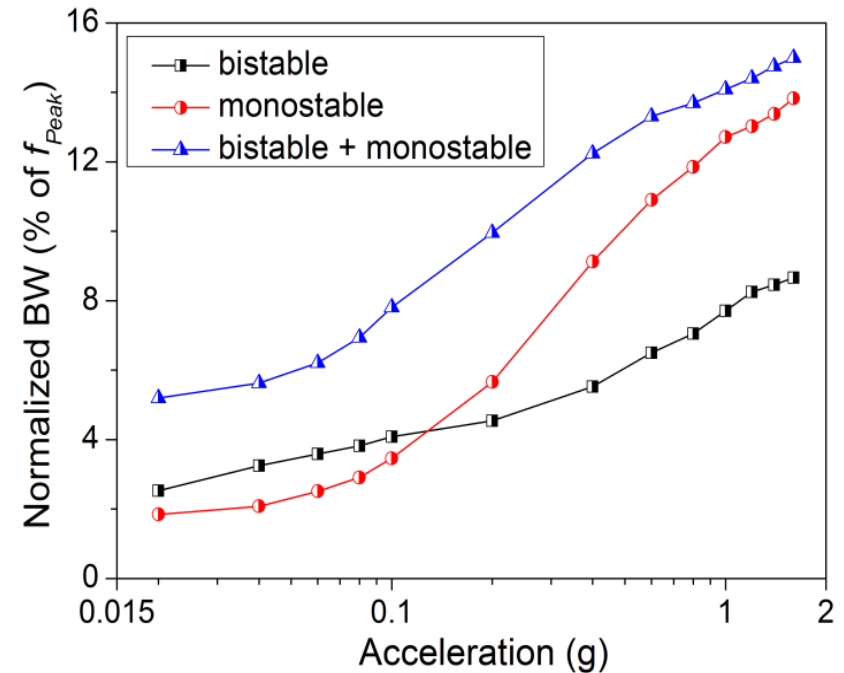
- Monostable and Bistable nonlinearity combined in a single device
- Engineered potential energy for better performance

Combined Multiple Nonlinearity (II)

Load Power vs Frequency Responses:



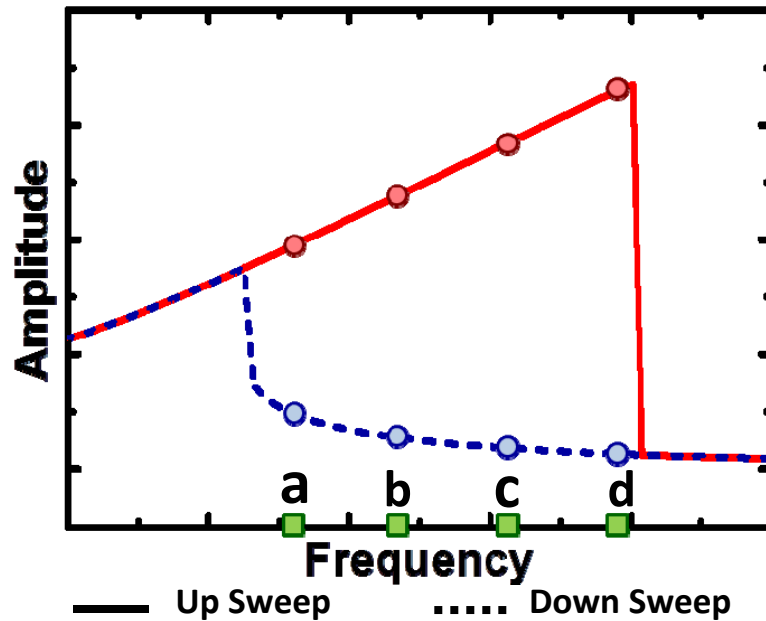
Normalized Bandwidth variation:



- Improvement in the low frequency region
- Two times Bandwidth compared to individual mono/bistable systems
- Major advantage in low acceleration applications

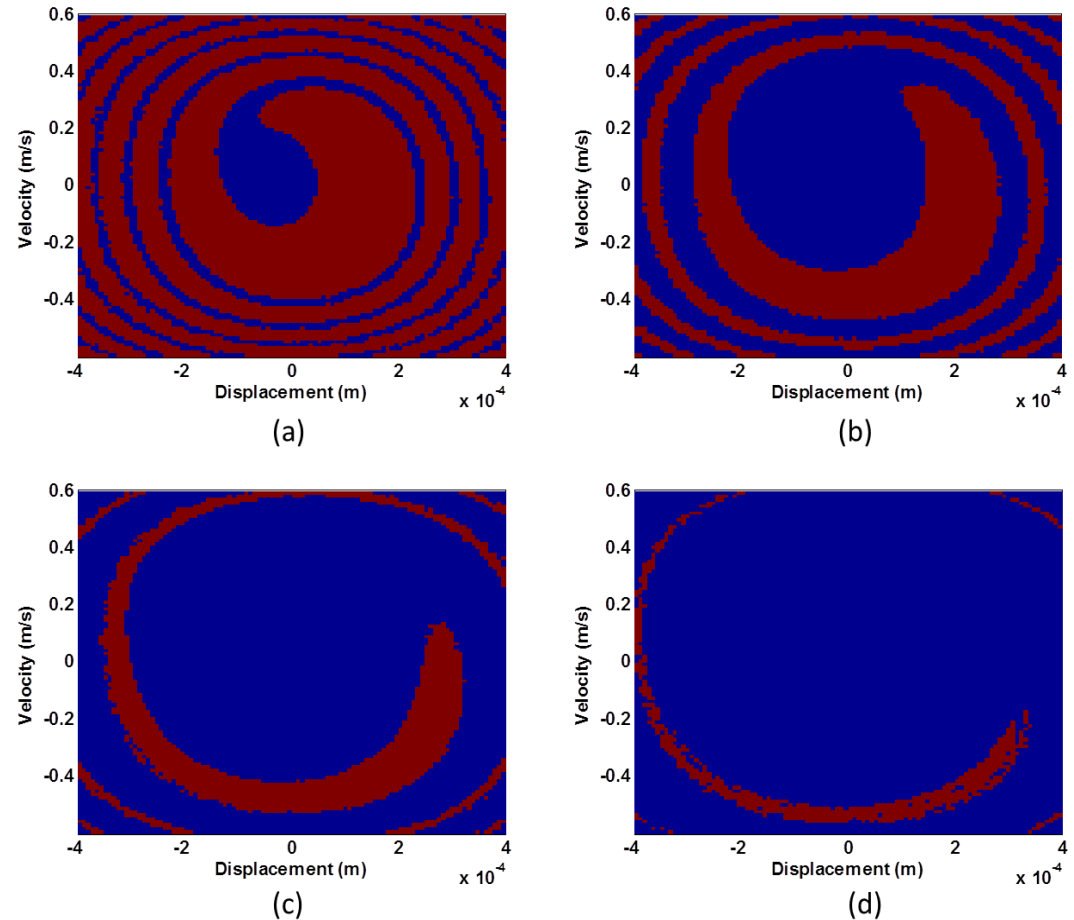
Nonlinear Hysteresis

Frequency Domain Response



- Multiple steady state solutions - Hysteresis
- How to operate in the frequency varying environment?

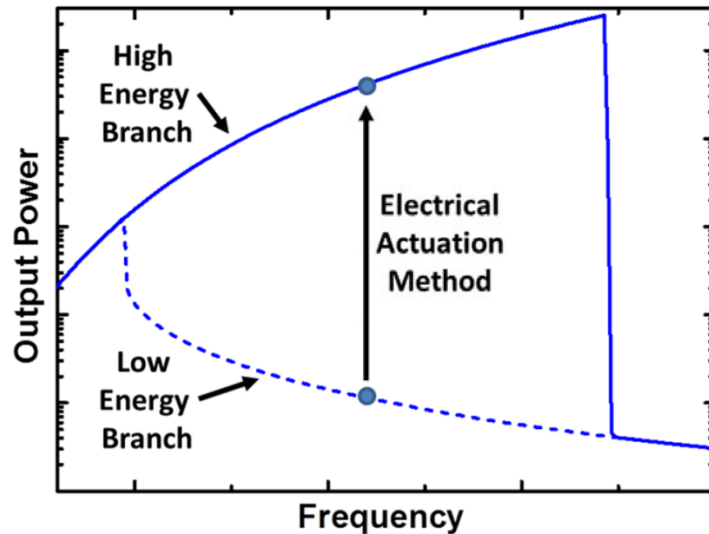
Basin of Attraction Plots within Hysteresis



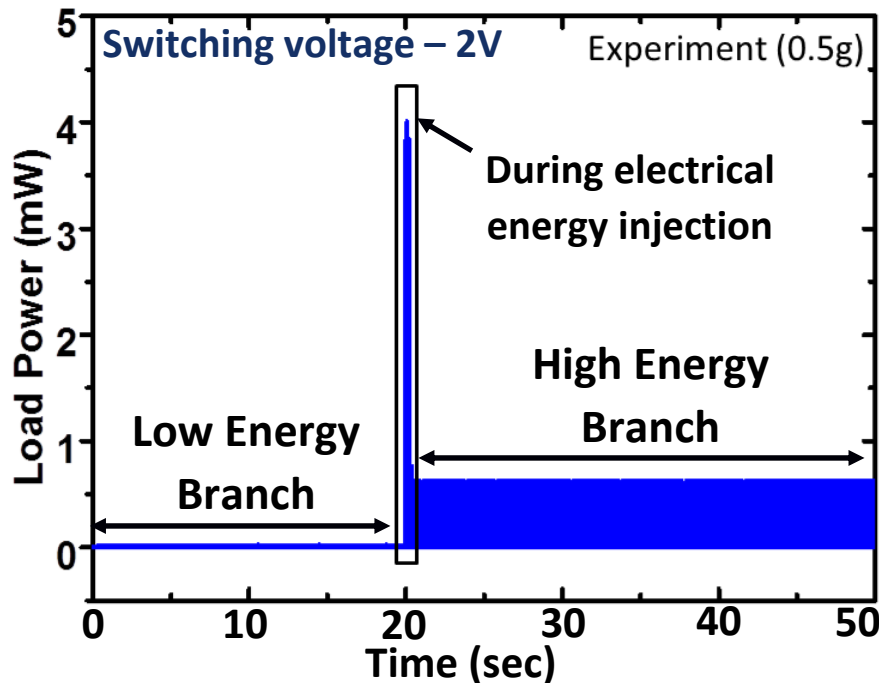
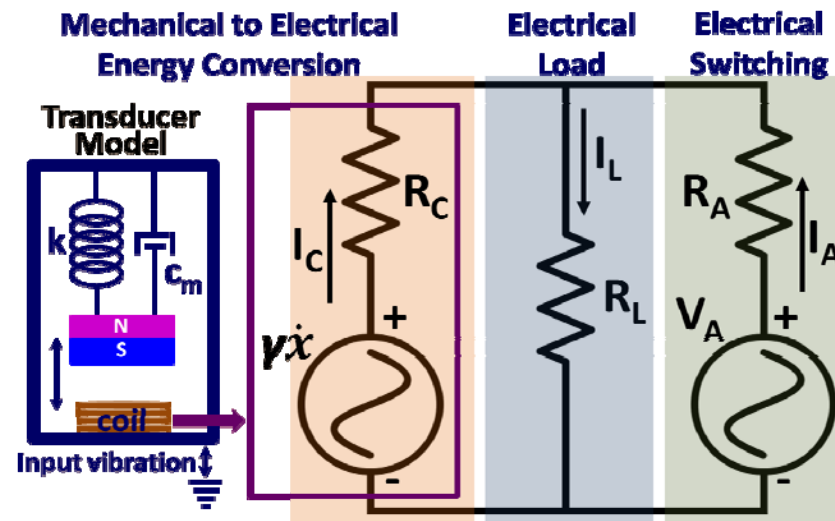
Blue – Low Energy Red – High Energy

Surfing the High Energy Branch (I)

Concept



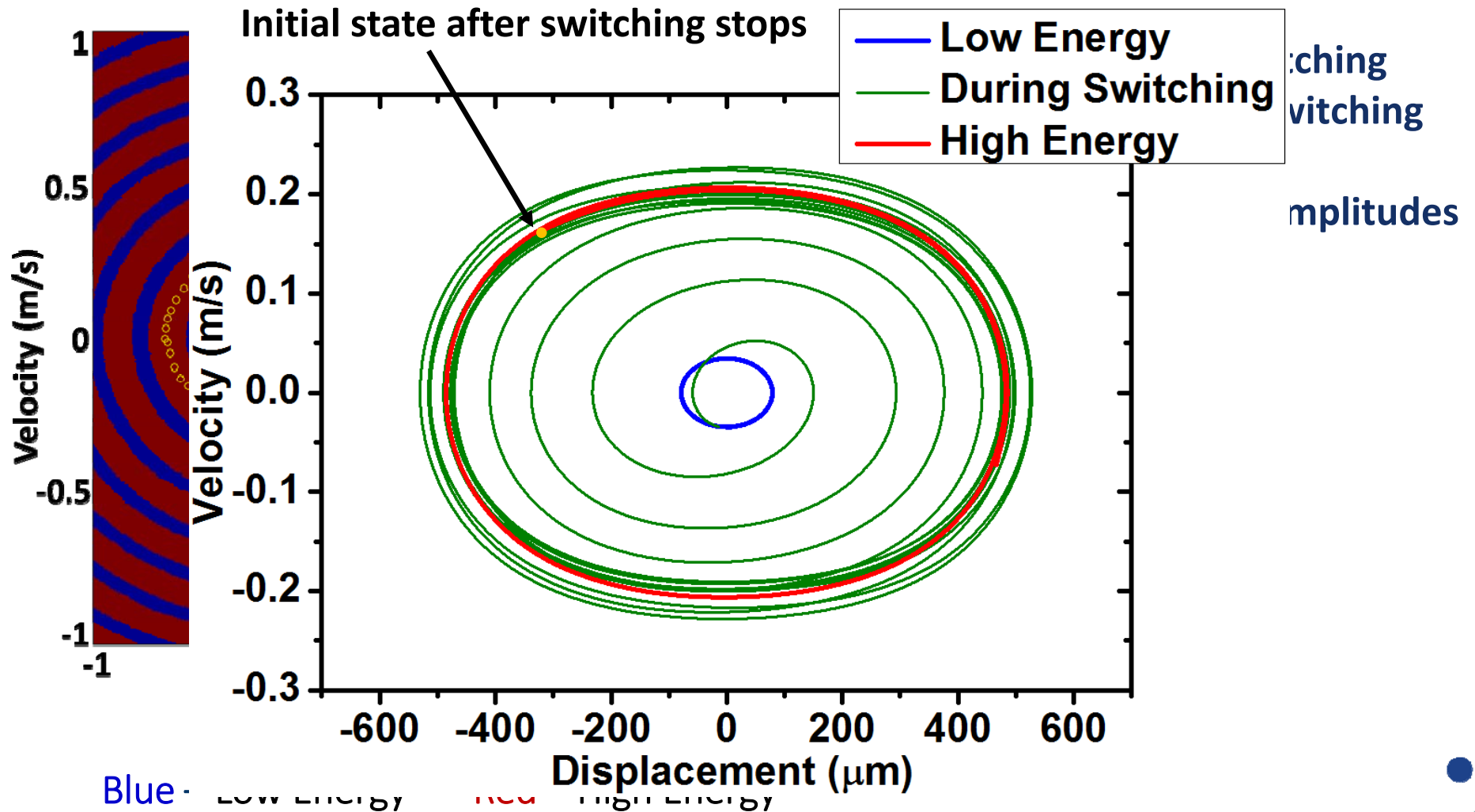
Implementation



- Injected energy switches the state from actual to desired attractor
- Maintains steady state – without continuous energy input

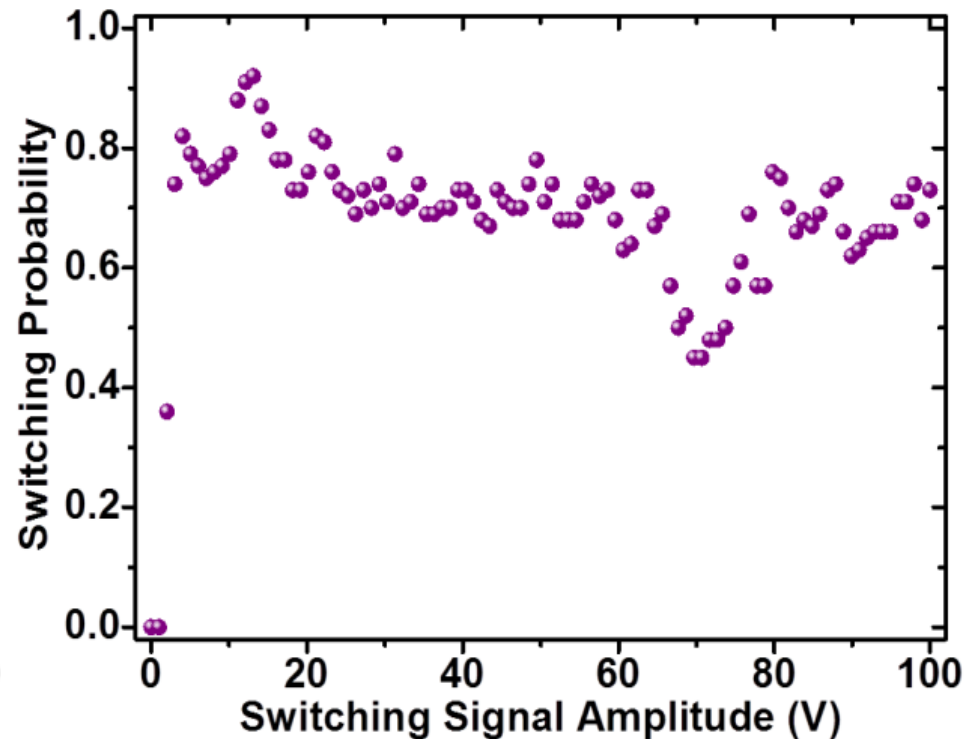
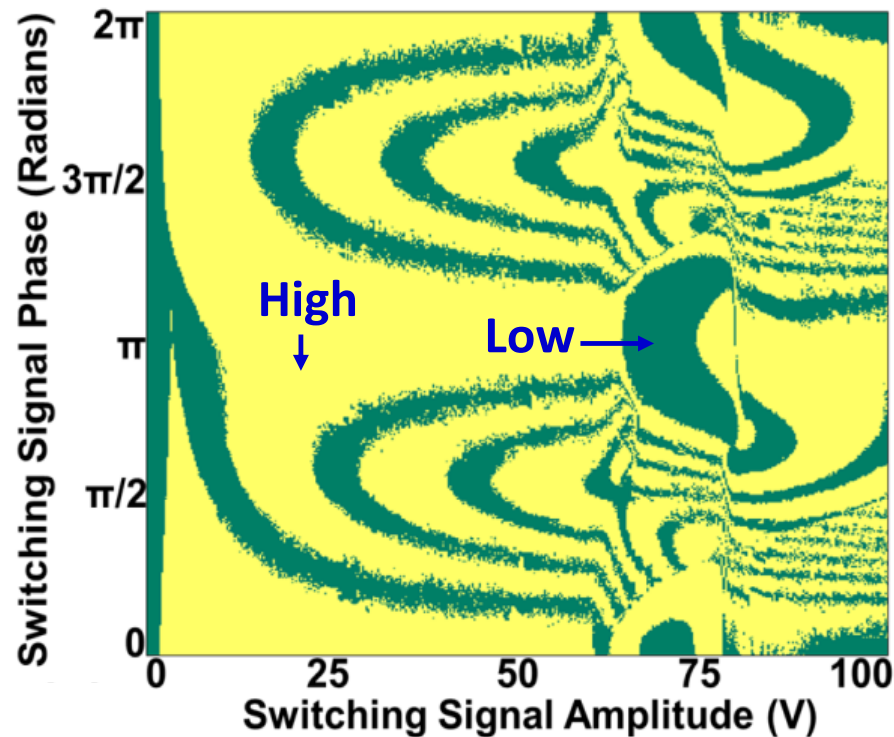
Successful/Unsuccessful Switching

Phase Space Diagram



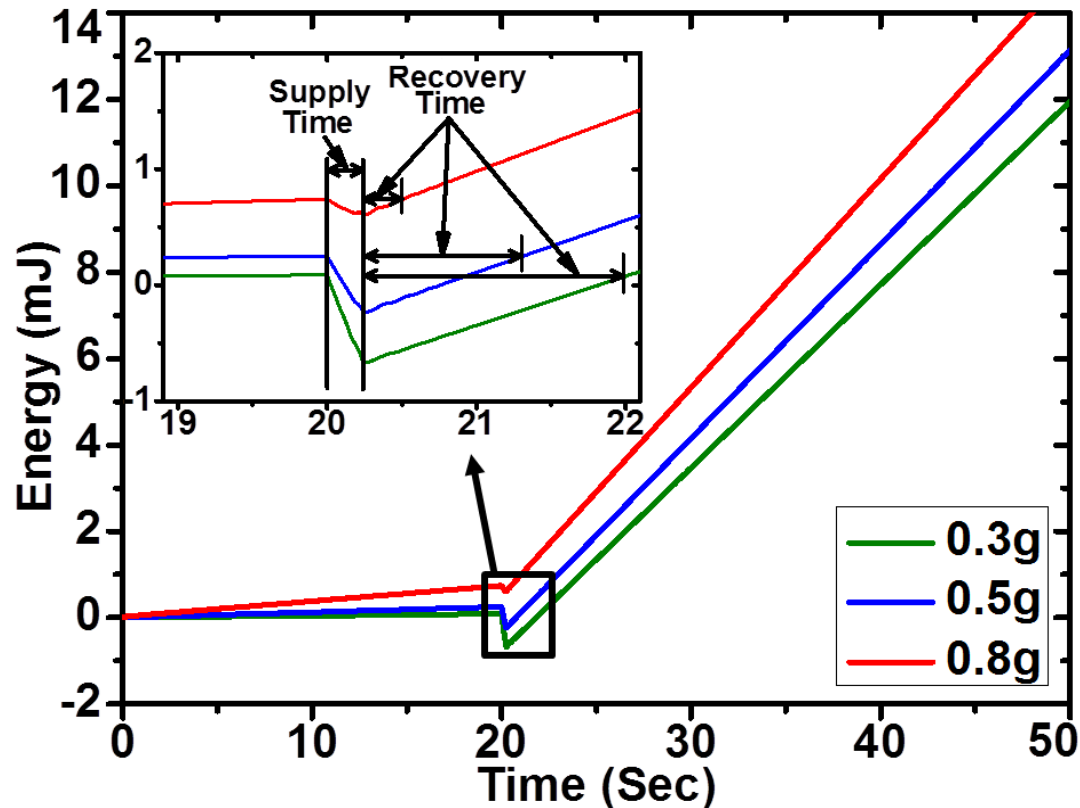
Surfing the High Energy Branch (II)

Probabilistic Study on Switching Mechanism



Surfing the High Energy Branch (III)

Evolution of Net Electrical Energy



E_0 - Energy to apply switching signal once

P_s - Probability of successful switching in first attempt

E_T - Total energy spent to switch the state

k - Number of attempts

$$E_T = P_s E_0 \sum_{k=1}^{\infty} k(1 - P_s)^{k-1} = \frac{E_0}{P_s}$$

As $P_s \sim 0.8$, E_T - not very high

Remarks

Conclusions:

- **Stretching nonlinearity** with modal overlap – ideal for MEMS VEH devices
- **Bistable-Monostable VEH** combines the beneficial features of bistable and monostable nonlinearities, and delivers better spectral response than both.
- **Multi stability** – major challenge in nonlinear oscillator based applications
- **Switching mechanism in nonlinear VEH** – Huge improvement energy conversion efficiency in a real application environment.
- **Switching mechanism** - independent of device scale or transduction methods

Future Work:

- **Automatize the Switching:** Self-controlled feedback loop development along with the power management circuit

Acknowledgement:

SFI PI grant award (2012)- MEMS vibrational energy harvesting – 11/PI/1201

People involved in this work:



Dr. Andreas Amann



Dr. Pranay Podder



Dr. Peter Constantinou



Ms. Kankana Paul

Tyndall - Speciality Products & Service (SP&S) group

Thank You

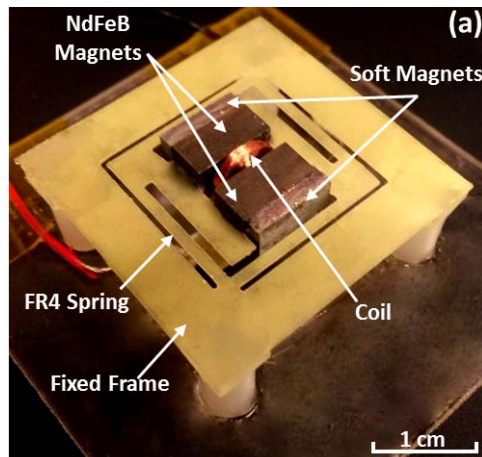
Contact: dhiman.mallick@tyndall.ie

Low Frequency Operation (Meso-scale)

Advantages of FR4 (Flame Retardant 4):

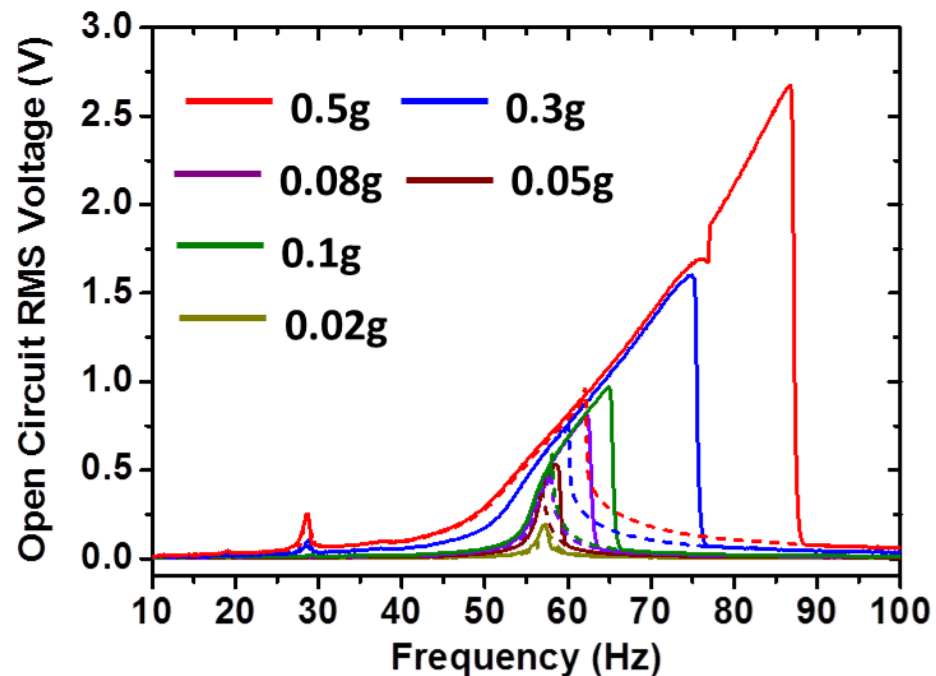
- Standard PCB material
- Low Young's Modulus (21 GPa) – Useful for low frequency applications.
- Low cost !!

Meso-scale Prototype



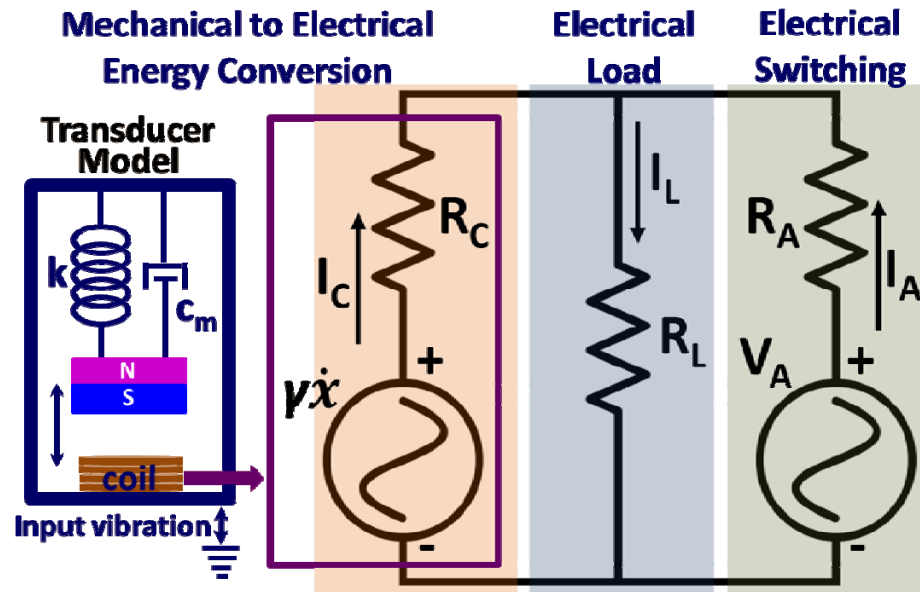
Volume = 2.65 cm^3

Mass = 3 gm



- **Bandwidth (BW) – 10 Hz @ 0.5g**
- **Maximum Power ~ 0.5 mW @ 0.5g**

Modelling of the Switching Scheme



$$V_A(t) = \begin{cases} V_{OA} \sin(2\pi f_A t) & \text{for } t_i \leq t \leq t_f \\ 0 & \text{Otherwise} \end{cases}$$

V_{OA} – Amplitude

f_A – frequency

t_i and t_f - starting and ending times of the switching period

The net current I_C through the coil:
$$I_C(\dot{x}, t) = I_L - I_A = \frac{\gamma\dot{x}}{R_C} - \frac{R_L}{R_C} \cdot \frac{V_A(t)R_C + \gamma\dot{x}R_A}{R_C R_A + R_A R_L + R_L R_C}$$

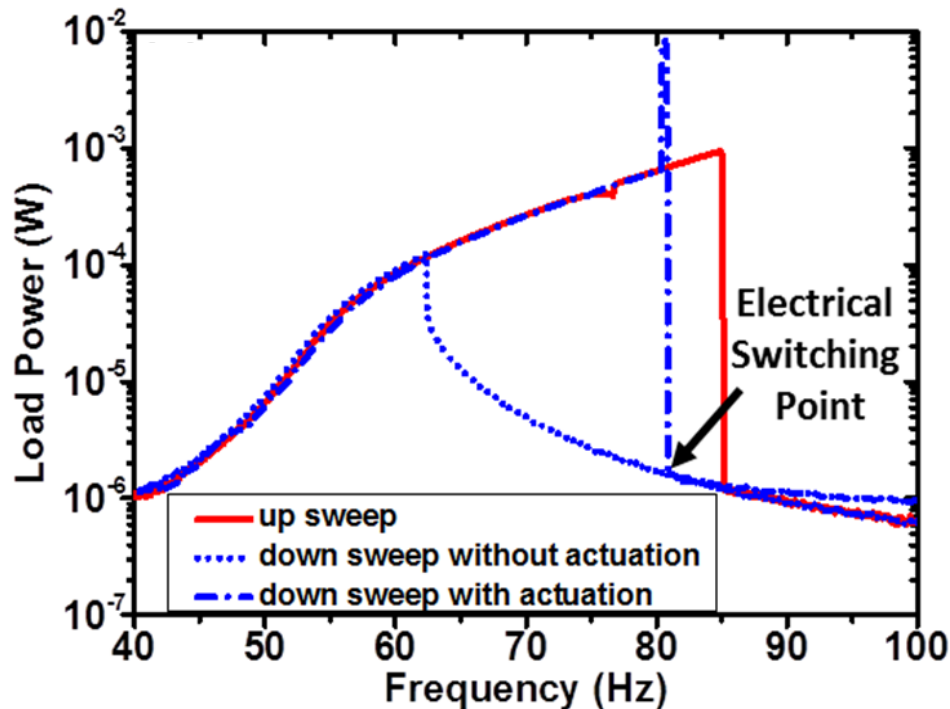
The coupled electromechanical equation of motion of the oscillator

$$m\ddot{x} + 2m\rho\omega_n\dot{x} + kx + k_n x^3 + \gamma I_C = -m\ddot{z}$$

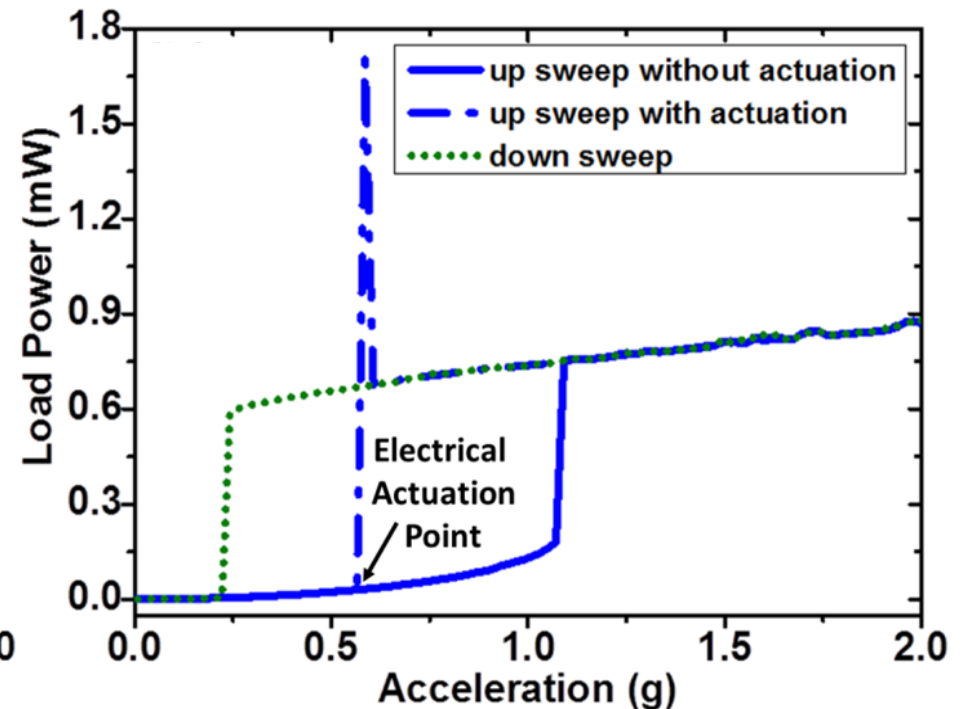
Surfing the High Energy Branch (II)

Altering the well established
frequency/amplitude scan responses

Fixed Acceleration – 0.5g

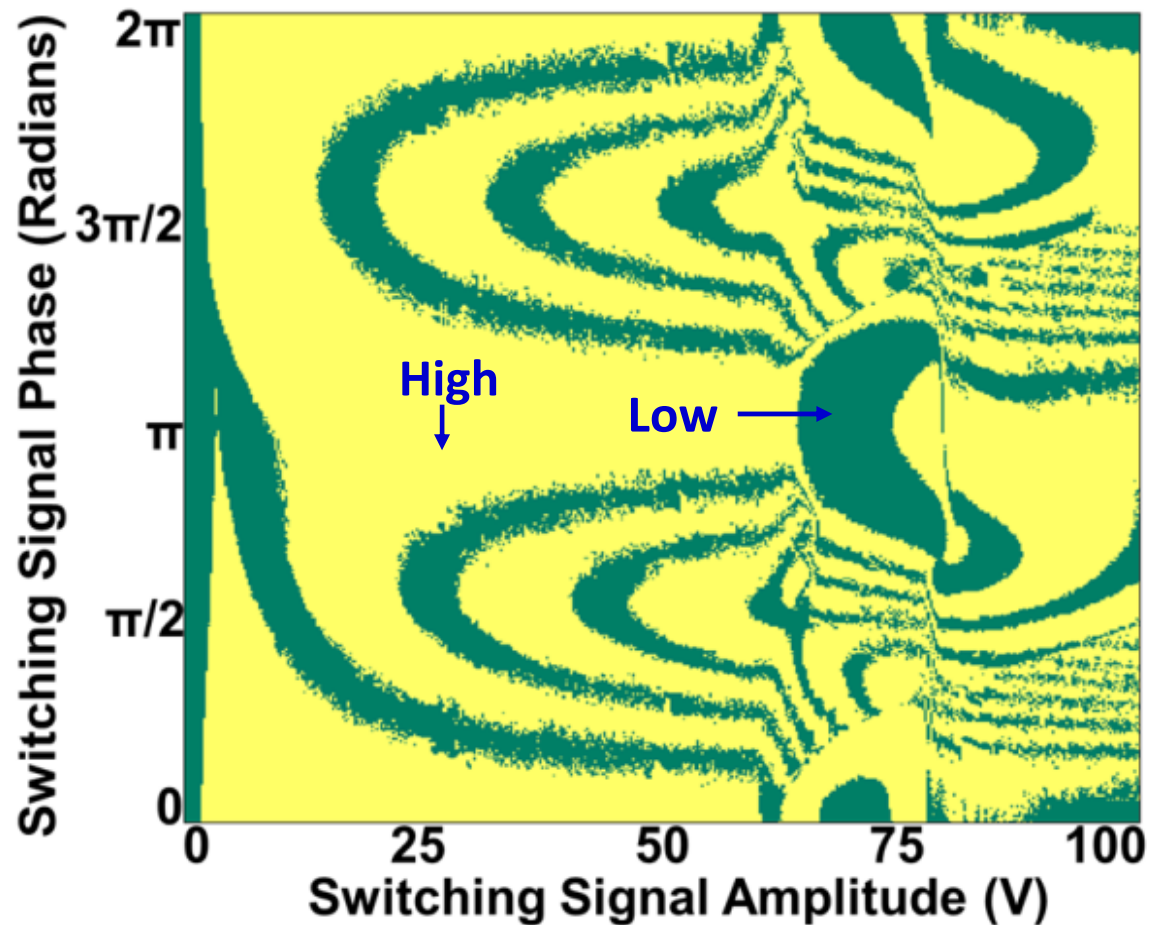


Fixed Frequency – 70 Hz



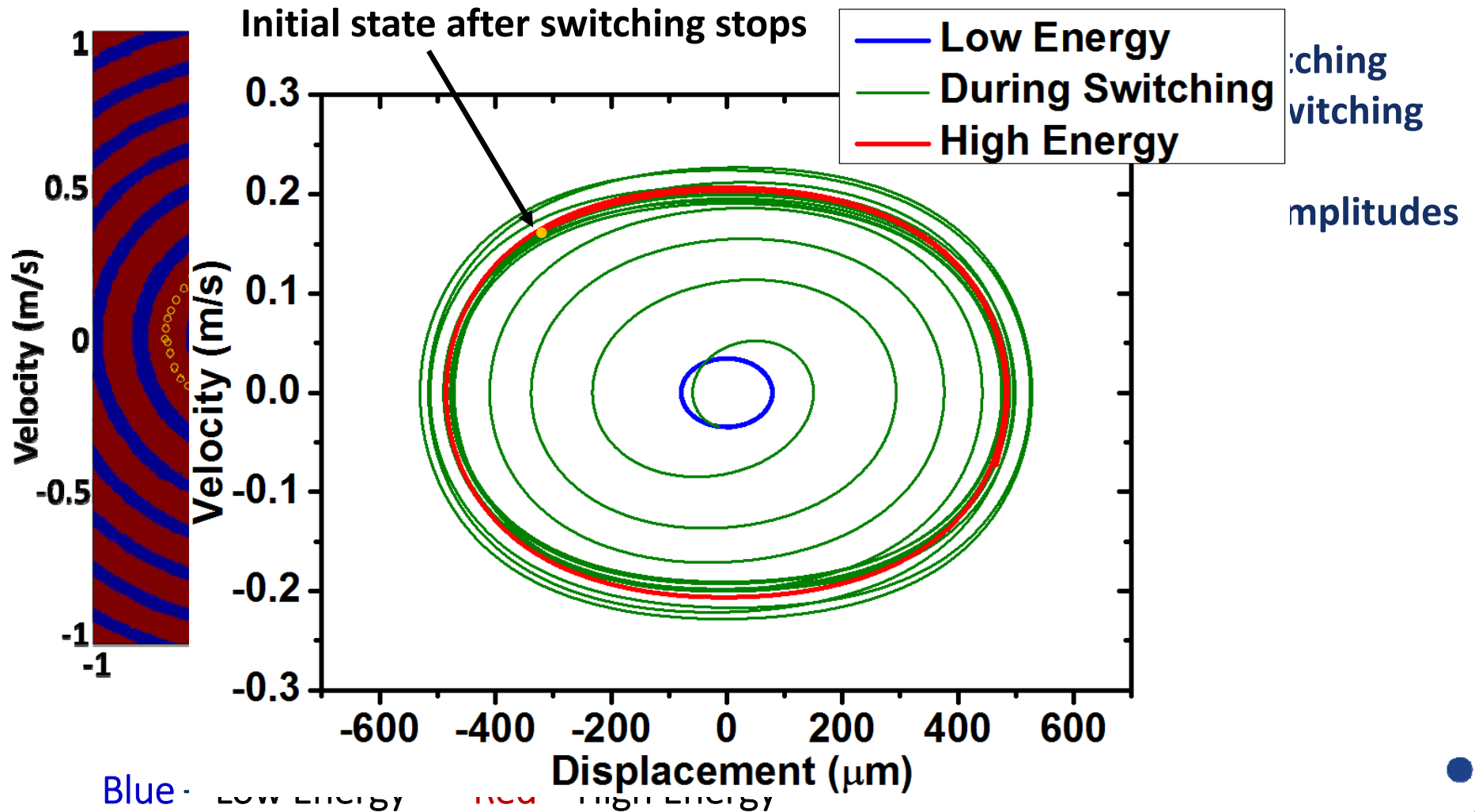
Surfing the High Energy Branch (II)

Probabilistic Study on Switching Mechanism



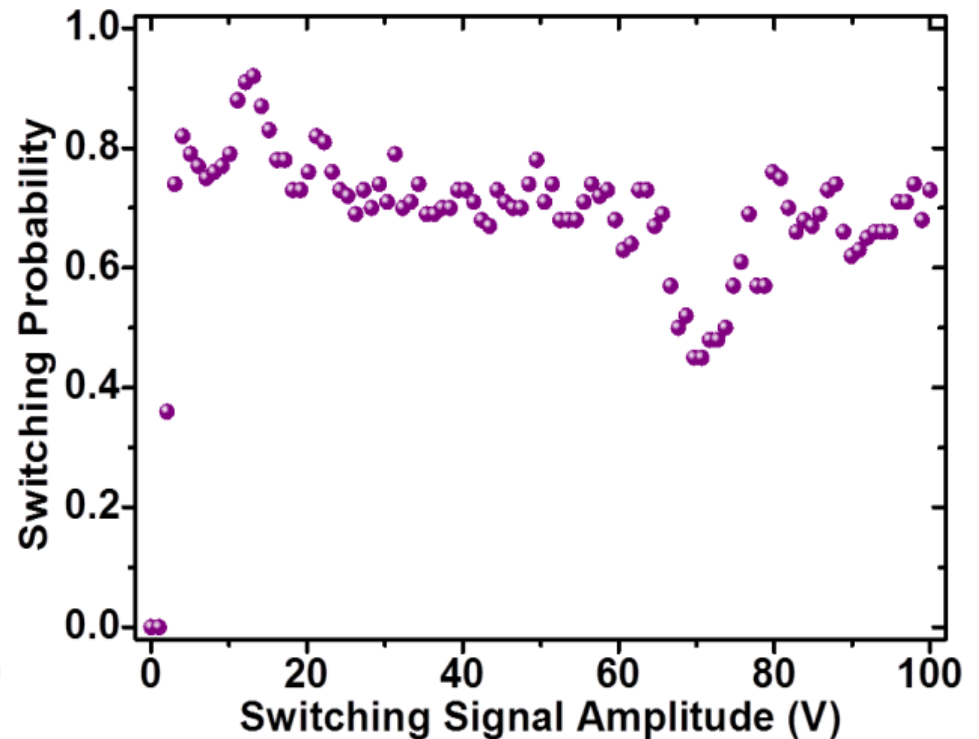
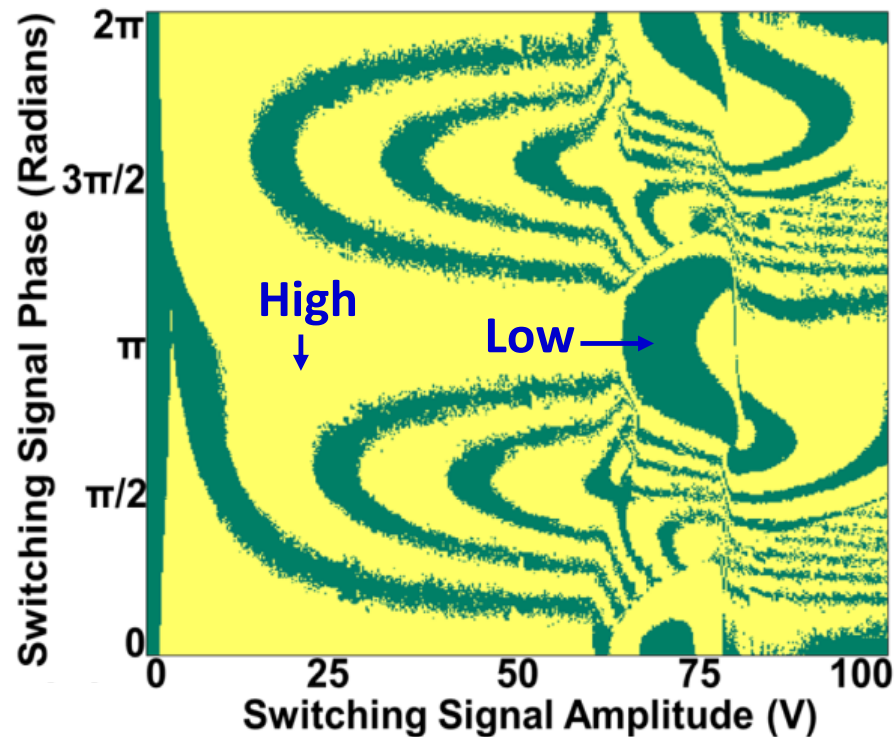
Successful/Unsuccessful Switching

Phase Space Diagram



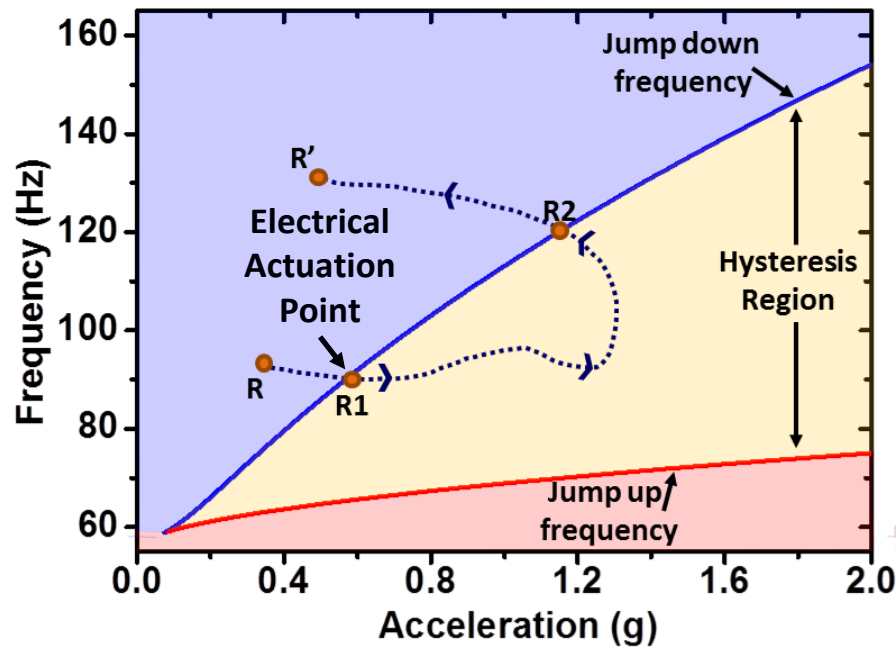
Surfing the High Energy Branch (II)

Probabilistic Study on Switching Mechanism

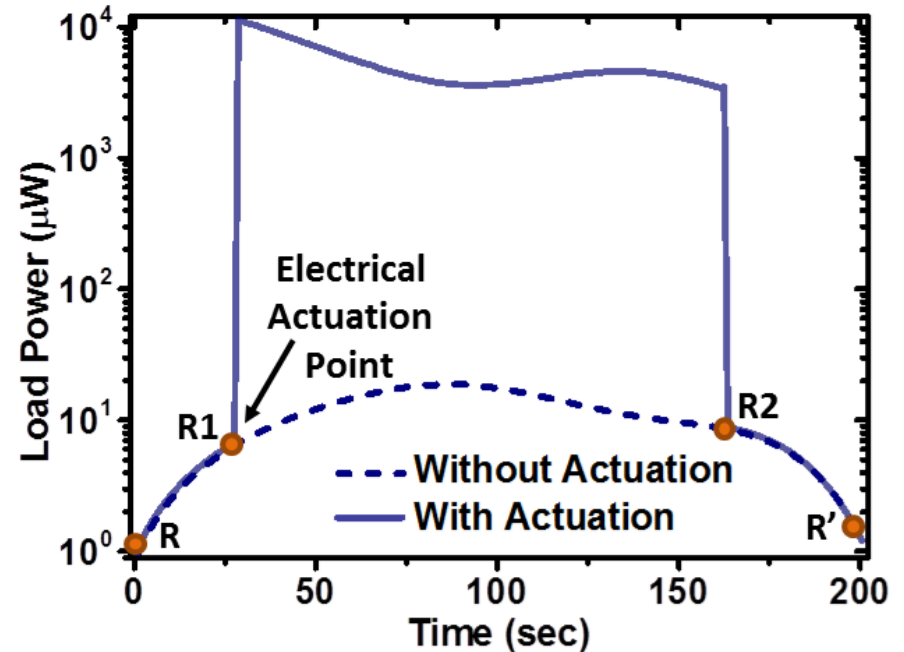


Surfing the High Energy Branch (III)

Response under randomly varying vibration

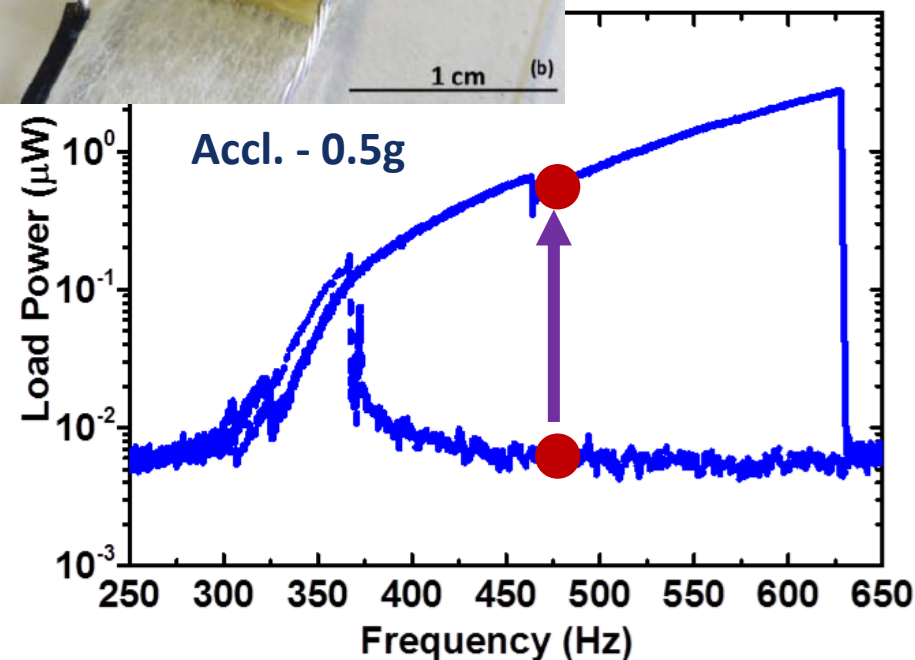
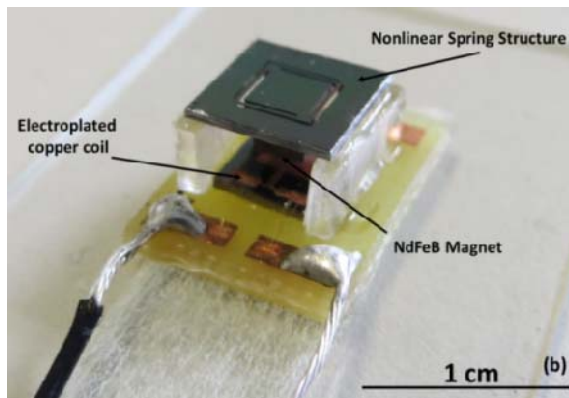


$R \rightarrow R'$: frequency/amplitude varying input excitation

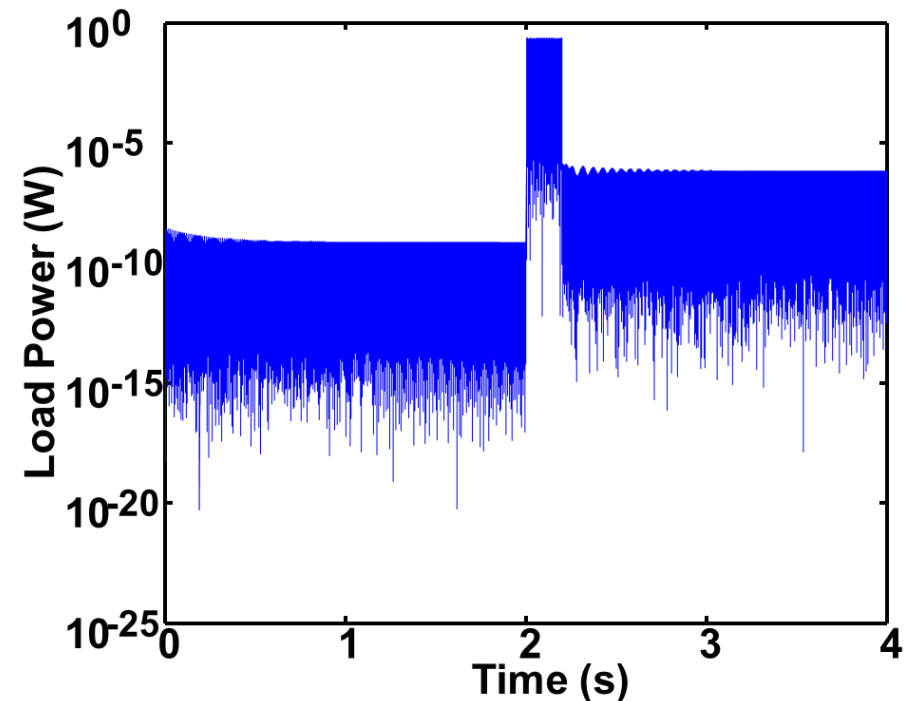


Increase in harvesting efficiency:
340 times

Electrical Switching in MEMS EM VEH



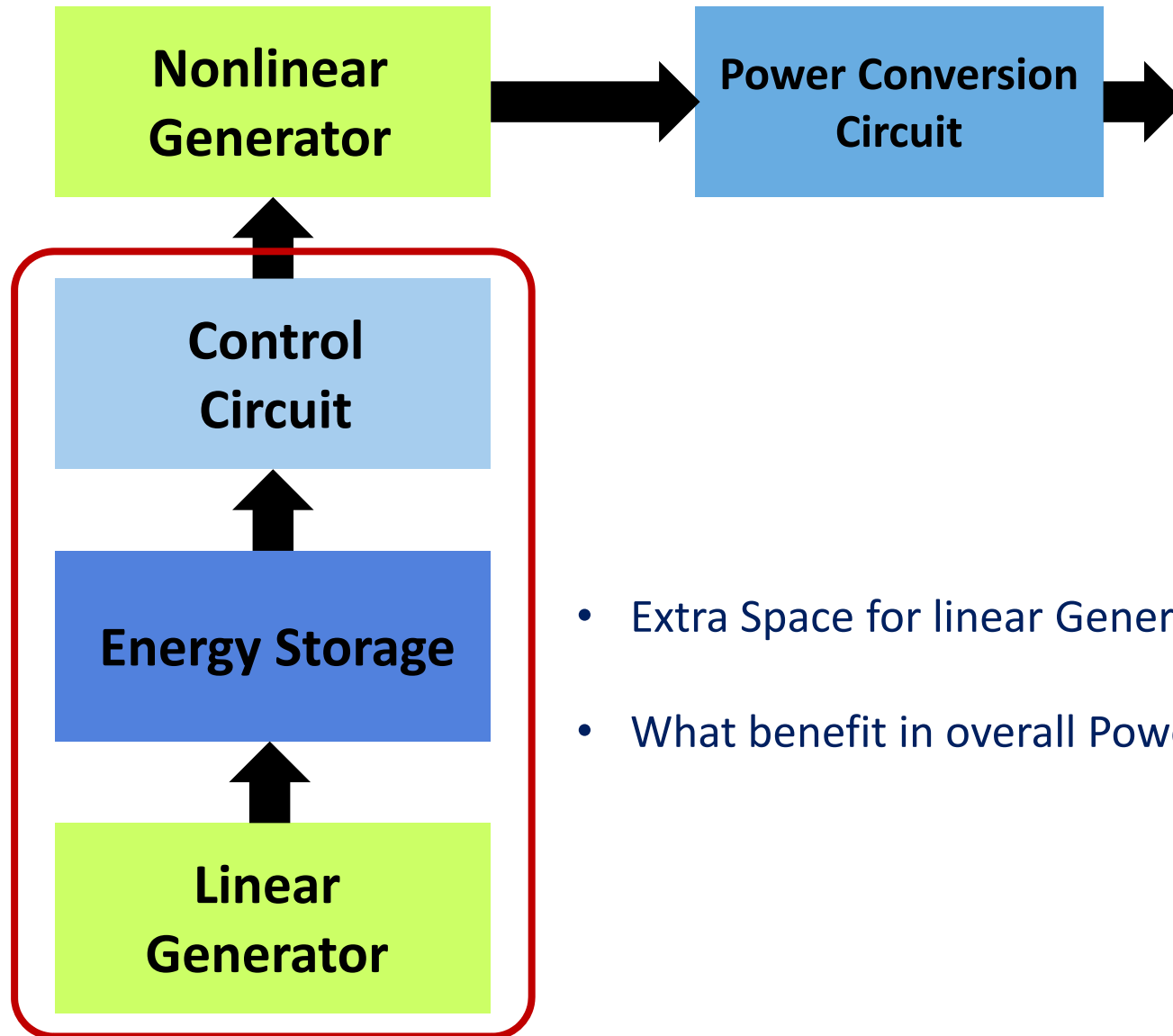
Switching voltage – 8V



- Energy gap between High and Low energy branches is low
- Electromechanical coupling is very low as well
- Inefficient transfer of energy

Self-switching/Automatic Operation

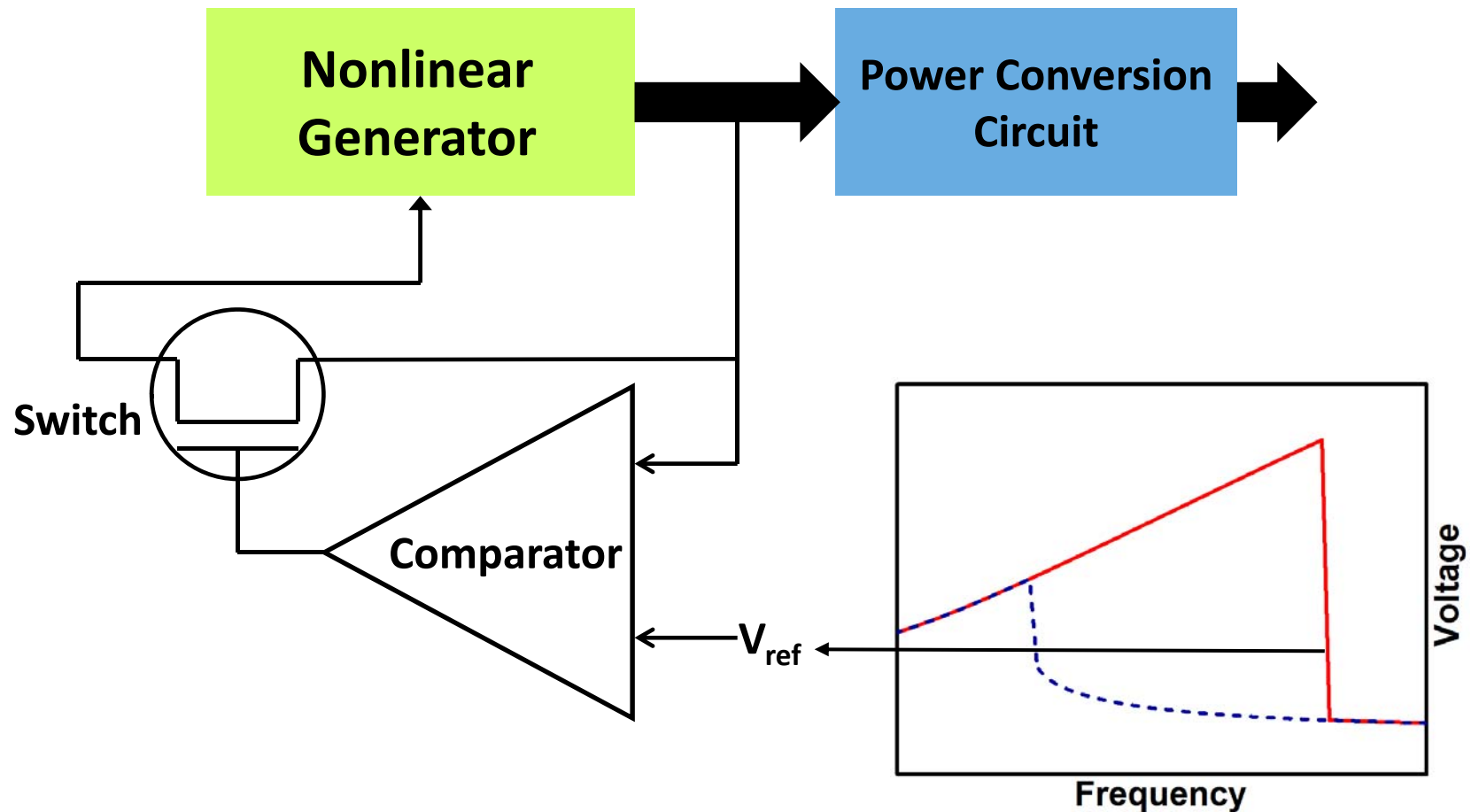
Proposed Scheme I



- Extra Space for linear Generator
- What benefit in overall Power Density?

Self-switching/Automatic Operation (II)

Proposed Scheme II



- Power loss even when there is no multi-stability